Contrôle 1, corrigé

Ex. 1.

$$\begin{cases} x-y+z=-1 \\ x-y=0 \end{cases} \quad \begin{array}{c} \\ \text{II}-\text{I} \end{array} \begin{cases} \boxed{x}-\overset{\text{libre}}{y}+z=-1 \\ \boxed{-z}=1 \end{cases} \quad \begin{cases} x-y=0 \\ z=-1 \end{cases} \quad \begin{cases} x=y \\ z=-1 \end{cases}$$

les solutions sont données par la droite $\begin{cases} x=t \\ y=t \\ z=-1 \end{cases}$ soit $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

Ex. 2.

$$\begin{cases} x+dz=2 \\ x+y=0 \\ -x+y=1 \end{cases} \quad \begin{array}{c} \text{II} \\ \text{I} \\ \text{III} \end{array} \begin{cases} x+y=0 \\ x+dz=2 \\ -x+y=1 \end{cases} \quad \begin{array}{c} \\ \text{II}-\text{I} \\ \text{III}+\text{I} \end{array} \begin{cases} \boxed{x}+y=0 \\ -y+dz=2 \\ 2y=1 \end{cases}$$

$$\begin{array}{c} \\ \text{III} \\ \text{II} \end{array} \begin{cases} x+y=0 \\ 2y=1 \\ -y+dz=2 \end{cases} \quad \begin{array}{c} \\ \frac{1}{2}\text{II} \\ \frac{1}{2}\text{II}+\text{III} \end{array} \begin{cases} \boxed{x}+y=0 \\ \boxed{y}=\frac{1}{2} \\ \boxed{dz}=\frac{5}{2} \end{cases}$$

si $d\neq 0$: une seule solution :

$$\begin{cases} x=-y=-\frac{1}{2} \\ y=\frac{1}{2} \\ z=\frac{1}{d}\cdot\frac{5}{2} \end{cases} \qquad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{d}\cdot\frac{5}{2} \end{pmatrix}$$

si $d=0$: $\begin{cases} x+y=0 \\ y=\frac{1}{2} \\ 0=\frac{5}{2} \end{cases}$ $\Rightarrow$ aucune solution.

Ex. 3.

$$A=\underset{2\times 4}{\begin{pmatrix} 2 & 1 & 3 & 4 \\ 0 & -1 & -1 & 0 \end{pmatrix}} \qquad B=\underset{3\times 2}{\begin{pmatrix} 1 & 2 \\ 0 & -1 \\ 1 & 0 \end{pmatrix}}$$

AB n'est pas possible

BA est possible : $(3\times 2)\cdot(2\times 4)=(3\times 4)$

$$BA = \begin{pmatrix} 1 & 2 \\ 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 1 & 3 & 4 \\ 0 & -1 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & -1 & 1 & 4 \\ 0 & 1 & 1 & 0 \\ 2 & 1 & 3 & 4 \end{pmatrix}$$

<u>Ex 4.</u>

$$\left(\underbrace{\begin{matrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{matrix}}_{A} \;\middle|\; \underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{\mathbb{1}} \right) \Rightarrow \begin{array}{c} \mathrm{I} \\ \mathrm{III} \\ \mathrm{II} \end{array} \left(\begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{matrix} \;\middle|\; \begin{matrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{matrix}\right)$$

$$\underset{\mathrm{III}-\mathrm{I}}{\Rightarrow} \left(\begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \end{matrix} \;\middle|\; \begin{matrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{matrix}\right) \underset{\mathrm{III}-\mathrm{II}}{\Rightarrow} \left(\begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{matrix} \;\middle|\; \begin{matrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & -1 \end{matrix}\right)$$

$$\underset{-\mathrm{III}}{\Rightarrow} \left(\begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \;\middle|\; \begin{matrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 1 \end{matrix}\right) \underset{\mathrm{I}-\mathrm{III}}{\Rightarrow} \left(\underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{\mathbb{1}} \;\middle|\; \underbrace{\begin{matrix} 0 & 1 & -1 \\ 0 & 0 & 1 \\ 1 & -1 & 1 \end{matrix}}_{A^{-1}}\right)$$

vérification $\begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & -1 \\ 0 & 0 & 1 \\ 1 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ok!