$$A = \begin{pmatrix} 2 & -1 & 0 \\ -2 & 1 & -2 \\ 1 & 1 & 3 \end{pmatrix}$$

$$P = \begin{array}{c} \begin{array}{ccc} \varepsilon_1 & \varepsilon_2 & \varepsilon_3 \end{array} \\ \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ -1 & -1 & 0 \end{pmatrix} \end{array}$$

$\det P = 0 + 1 - 1 - 0 - 1 - 0 = 1 \Rightarrow P$ inversible

$\underline{B' \text{ base}}$

ou sinon :

$$P \sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{rk } P = 3 \Rightarrow B' \text{ base}$$

noyau et im. de A :

$$A \sim \begin{array}{c} \text{III} \\ \text{I}-2\text{III} \\ \text{II}+2\text{III} \end{array} \begin{pmatrix} 1 & 1 & 3 \\ 0 & -3 & -6 \\ 0 & 3 & 4 \end{pmatrix} \sim \begin{array}{c} \\ \\ \text{III}+\text{II} \end{array} \begin{pmatrix} 1 & 1 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & -2 \end{pmatrix} \Rightarrow \text{rk} A = 3 \Rightarrow \text{im } A = \mathbb{R}^3$$

$\ker A = 0.$

$$f(\varepsilon_1) = A \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \varepsilon_1$$

$$f(\varepsilon_2) = A \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix} = 2\varepsilon_1$$

$$f(\varepsilon_3) = A \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ 0 \end{pmatrix} = 3\varepsilon_1$$

$$\Rightarrow \text{Mat}_{B'}(f) = \begin{array}{c} \begin{array}{ccc} \varepsilon_1 & \varepsilon_2 & \varepsilon_3 \end{array} \\ \begin{pmatrix} 1 & & \\ & 2 & \\ & & 3 \end{pmatrix} \end{array} = D$$

$$P^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -2 \\ 1 & 0 & 1 \end{pmatrix}$$

$$P^{-1}AP = \begin{pmatrix} 1 & & \\ & 2 & \\ & & 3 \end{pmatrix} = D$$

alors $A = PDP^{-1}$ et

$$A^n = (PDP^{-1})^n = \underbrace{PD\cancel{P^{-1}P}D\cancel{P^{-1}P}DP^{-1}}_{n} = PD^nP^{-1} = P \begin{pmatrix} 1^n & & \\ & 2^n & \\ & & 3^n \end{pmatrix} P^{-1} = \ldots$$