Special Genera of Hermitian Lattices and Applications to HAWK

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Guilhem Mureau

Inria de Bordeaux, IMB, France

The Lattice Isomorphism Problem (LIP)

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$\mathcal{L} := \{\sum_i x_i \mathbf{b_i} \mid x_i \in \mathbb{Z}\}$ is a **lattice** in $\mathbb{R}^n$ and $\mathbf{B} = (\mathbf{b_1}| \cdots |\mathbf{b_n})$ is a **basis** of $\mathcal{L}$.

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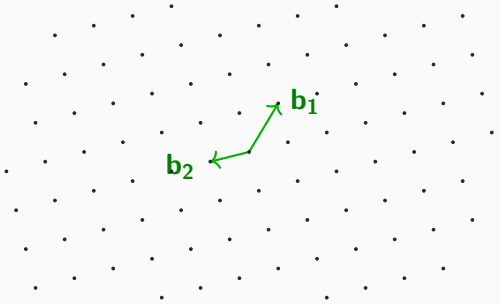

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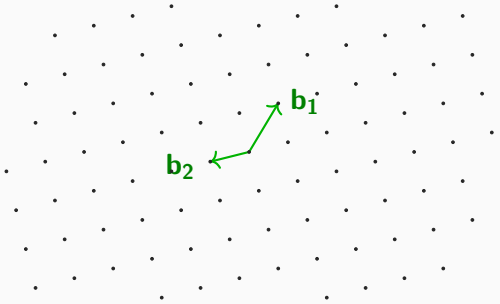

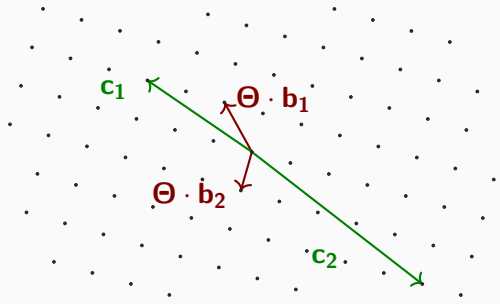

Lattices $\mathcal{L}_1$ and $\mathcal{L}_2$ are **isomorphic** if $\mathcal{L}_2 = \mathbf{\Theta}(\mathcal{L}_1)$ for some $\mathbf{\Theta} \in \mathcal{O}_n(\mathbb{R})$ orthogonal.

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Use in cryptography :

1. Ducas & van Woerden (2021): primitives based on decision LIP.
2. Ducas *et. al.* (2022): signature scheme **HAWK**, based on search **module**-LIP.

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van Gent & van Woerden (2025): reduce search module-LIP to **decision** module-LIP.
$\rightsquigarrow$ HAWK reduces to several instances of decision module-LIP.

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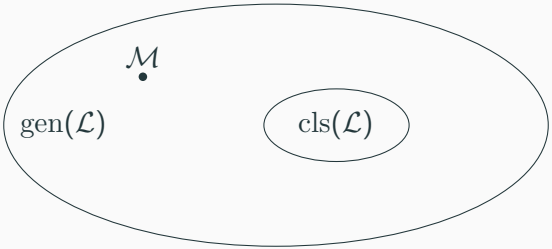

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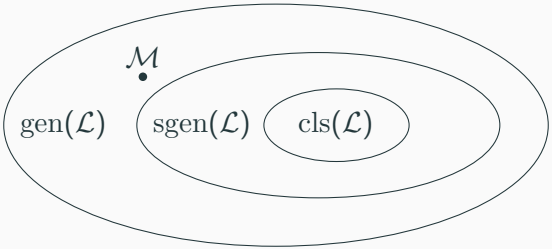

Contribution: The **special genus** gives an intermediate classification.
Moreover $\mathcal{M} \in \mathrm{sgen}(\mathcal{L})$ can be **tested efficiently**, for a large family of module lattices.

Background on number theory (1)

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Its **ring of integers** is $\mathbb{Z}_K = \mathbb{Z}[X]/(X^n + 1)$. $\exists$ a **complex conjugation** $a \mapsto \overline{a}$ on K.

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K **embeds** into larger fields. Two types of embeddings:

1. **Complex**: $K \hookrightarrow \mathbb{C}$, by sending X to a root of $X^n + 1$ in $\mathbb{C}$.
2. **Local**: $K \hookrightarrow K_\mathfrak{p}$, for any prime ideal $\mathfrak{p}$.

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For any prime ideal $\mathfrak{p}$, a module lattice $\mathcal{L} \subset K^\ell$ extends to $\mathcal{L}_\mathfrak{p} \subset K_\mathfrak{p}^\ell$ via $K \hookrightarrow K_\mathfrak{p}$.
It is **easy** to check if $\mathcal{L}_\mathfrak{p}$ and $\mathcal{M}_\mathfrak{p}$ are **locally** isomorphic at $\mathfrak{p}$, *i.e.*, if there exists

$$\mathbf{\Theta}_\mathfrak{p} \in \mathcal{U}_\ell(K_\mathfrak{p}) \text{ s.t. } \mathcal{M}_\mathfrak{p} = \mathbf{\Theta}_\mathfrak{p}(\mathcal{L}_\mathfrak{p}), \quad \text{where } \mathcal{U}_\ell \text{ means unitary : } \overline{\mathbf{\Theta}}_\mathfrak{p}^T \cdot \mathbf{\Theta}_\mathfrak{p} = \mathbf{Id}.$$

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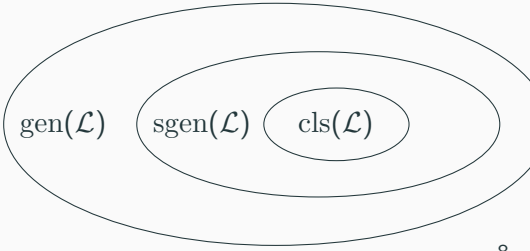
The genus of $\mathcal{L}$ contains the special genus of $\mathcal{L}$, itself containing the isomorphism class of $\mathcal{L}$.

There are **finitely many** classes in a genus.



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More generally it works for any $\mathcal{L}$ and $\mathcal{M} \in \mathrm{gen}(\mathcal{L})$ but it is harder to analyze.

Consequences and conclusion

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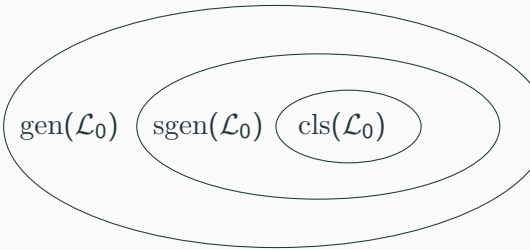
$\rightsquigarrow$ Still about $\approx 2^{800}$ classes in $\mathrm{sgen}(\mathcal{L}_0)$!



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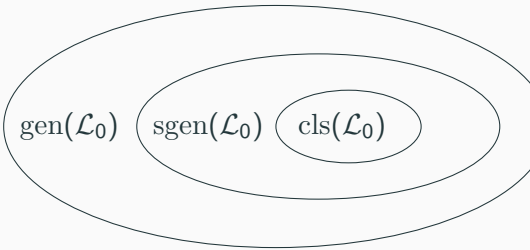

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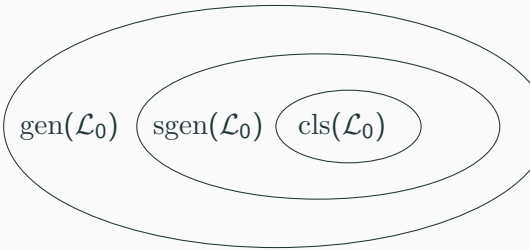

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- It is computable for several module lattices but has no practical impact on HAWK.

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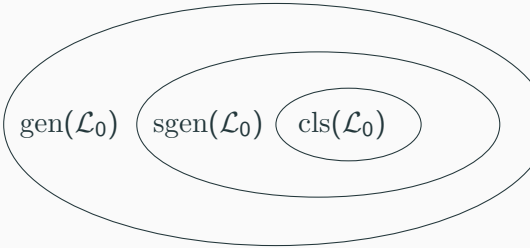

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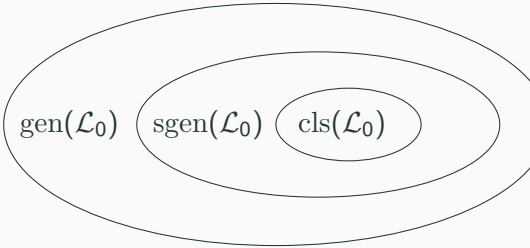
- The special genus is a finer invariant. To make decision module-LIP difficult, module lattices have to be chosen in the same special genus.
- It is computable for several module lattices but has no practical impact on HAWK.
- Open question: are there finer computable invariants?

Consequences and conclusion

Can we estimate the gain and the impact on HAWK? Set $\mathcal{L}_0 = \mathbb{Z}_K^2$ and $m = 512$.

$\#\{\text{isomorphism classes in } \mathrm{gen}(\mathcal{L}_0)\} \approx 2^{1000}$.
$\#\{\text{special genera in } \mathrm{gen}(\mathcal{L}_0)\} \approx 2^{200}$.

$\rightsquigarrow$ Still about $\approx 2^{800}$ classes in $\mathrm{sgen}(\mathcal{L}_0)$!



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Thank you for your attention!

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