2. CONTEXT, POSITION AND OBJECTIVES OF THE PROPOSAL

2.1. POSITION OF THE PROJECT

This project is in the area of mathematical physics. Its purpose is the rigorous mathematical analysis of physically relevant models arising from Quantum Field Theory (QFT). QFT provides a theoretical framework to study infinite dimensional quantum mechanical systems. Among the four fundamental interactions in nature, three (the electromagnetic, the weak and the strong force) can be formulated as quantum field theories, the common interpretation being that these interactions are mediated by the exchange of field particles (namely photons, intermediate vectors bosons and gluons respectively in the case of the electromagnetic, the weak and the strong force).

The approach to QFT that we will follow is based on second quantization; the particles of the quantized field(s) are associated with vectors in Fock space(s), and the energy of the system under consideration is described by a Hamiltonian (a self-adjoint operator) acting on a Hilbert space given in terms of the Fock space(s). The Hamiltonian generates a unitary dynamics representing the quantum evolution flow of the states of the system, and our aim is to reach a better understanding of this dynamics, in a mathematically precise way, for various QFT models.

The general feature of the mathematical questions we wish to address typically arises for models that have been intensively studied in the recent literature, describing charged non-relativistic quantum particles that interact with the quantized radiation field. Among these models, we can cite the so-called standard model of non-relativistic Quantum Electrodynamics (QED), which is obtained by quantizing the Newton equations of motion (for the charged particles) minimally coupled to the Maxwell equations (for the electromagnetic field), or caricatures of non-relativistic QED, for instance the spin-bosons model or the Nelson model. All these models can be seen as particular cases of an abstract class of QFT models that will be referred to as Pauli-Fierz models in this document.

In any case, the system is described by a Hilbert space of the form

$$\mathcal{H} = \mathcal{H}_{\text{part}} \otimes \mathcal{F},$$

and a Hamiltonian

$$H_g = H_{\text{part}} \otimes 1 + 1 \otimes H_{\text{field}} + gV$$

where H_{part} is the Hamiltonian of the charged particles acting in the Hilbert space $\mathcal{H}_{\text{part}}$ and H_{field} is the Hamiltonian of the free radiation field acting in the symmetric Fock space $\mathcal{F}$. For instance, in the case of the spin-bosons model, $\mathcal{H}_{\text{part}}$ is equal to C^2, and H_{part} is a Pauli matrix, while in the cases of the standard model of non-relativistic QED and the Nelson model, $\mathcal{H}_{\text{part}} = \mathrm{L}^2(\mathrm{R}^{3\mathrm{N}})$ (where N is the number of charged particles), and H_{part} is a Schrödinger operator. The symmetric Fock space, $\mathcal{F}$, is given by

$$\mathcal{F} = \mathrm{C} \oplus \bigoplus_{n=1}^{\infty} h^{\otimes_s^n},$$

where C is the set of complex numbers, $\mathfrak{h}$ is a L^2 space, and $\otimes_s^n$ denotes the symmetric n^{th} tensor product. The free field Hamiltonian H_{field} is the second quantization of the multiplication by the bosonic dispersion relation $\omega(k)$. In the sequel, the model will be called massless if $\omega(k) = |k|$, and massive if $\omega(k) = \sqrt{k^2 + m^2}$ with $m > 0$. Of course, physically, the massless case is of fundamental interest since the mass of photons, the field particles of QED, is equal to 0.

The last term appearing in the total Hamiltonian H_g represents the interaction between the charged particles and the field; the parameter g is a coupling constant and V is usually defined in terms of creation and annihilation operators. Let us mention that V is in general not a relatively compact perturbation of the free Hamiltonian H_0. Furthermore, in the massless case, the spectrum of H_0 usually contains eigenvalues sitting at the bottom of branches of essential spectrum. Hence, in particular, usual perturbation theory is inapplicable, and even in the small coupling regime (that is for g small enough), the study of H_g is highly non trivial.

If simplified to the extreme, the physical picture describing the evolution of this type of systems can be summed up as follows (at least in the massless case): any arbitrary initial state evolving according to the dynamics generated by H_g will eventually (as time goes to infinity) return to equilibrium (the ground state) by emitting radiation that takes the form of bosons escaping to infinity.
Justifying the previous physical picture in a precise way requires the study of different mathematical properties of H_g, including:

- The spectrum of H_g: one should be able to show the existence of a ground state for H_g (potentially in a non-Fock representation) and to establish the absolute continuity of the spectrum above the ground state energy. Hence in particular one should show that the excited states associated to the non-interacting Hamiltonian H_0 turn into resonances as the interaction gV is turned on.
- The asymptotic behavior of the dynamics generated by H_g; in other words, one has to develop a full scattering theory for H_g. One should be able to establish various propagation estimates showing how the field particles propagate, one should define and study wave operators, and one should prove asymptotic completeness in a suitable sense. Besides, determining the singularity of the resolvent of H_g near the resonances is crucial to apprehend the decay of metastable states.

In the last few years, these topics have been intensively studied by many authors in France and abroad. Under certain assumptions, (partial) answers to the questions raised above have been established. Apart from the members of the present project, we can mention the following internationally recognized experts (the list, of course, is not exhaustive): L. Amour, A. Arai, V. Bach, S. De Bièvre, J. Derezinski, J. Fröhlich, V. Georgescu, C. Gérard, B. Grébert, M. Griesemer, J.-C. Guillot, I. Herbst, F. Hiroshima, V. Jaksic, A. Joye, E. Lieb, M. Loss, J.S. Møller, F. Nier, C.-A. Pillet, B. Schlein, I.M. Sigal, E. Skibsted, H. Spohn… The issues we will work on within this project are at the heart of a very active domain of research, where substantial progresses are currently made.

We plan to work in two main directions:

- Although in the recent years a lot of effort was devoted to their study, several fundamental problems remain open for the (massless) Pauli-Fierz models related to non-relativistic QED introduced above. We will address some of these problems during the present project.
- The mathematical analysis of other QFT models describing important physical situations also started recently. We have in mind, in particular, the Nelson model on a static space-time (see [GHPS1,GHPS2,GHPS3]) which is an example of a model of QFT in curved space-time, and models of the weak interaction (see [BaG,ABFG]) and of relativistic QED (see [BDG]). Only a few basic properties have been obtained for these models up to now, and we plan to pursue their analysis.

In the next section, we give a brief overview of what is already known in the present context.

2.2. State of the art

In the sixties and seventies, a lot of research was devoted to the study of Hamiltonians of QFT; let us mention in particular a famous paper of Nelson, [Ne], the papers of Høegh-Krohn, [HK1,HK2,HK3], which can be seen as the first step of a rigorous construction of scattering theory in QFT, and the papers of Fröhlich, [Fr1,Fr2], on translation invariant models. More recently, starting with the seminal works of Bach, Fröhlich and Sigal, [BFS1,BFS2,BFS3], and Derezinski and Gérard, [DG1], spectral and scattering theories in QFT have become a very active area of research in mathematical physics.

In this section we begin with recalling known results for Hamiltonians of the form described in the previous section, next we briefly review the state of the art for other issues that we will consider.

Existence or non-existence of a ground state

A first question that naturally arises in the study of QFT models is the problem of the existence of a ground state. The latter justifies in some sense the stability of the model, since any initial state is expected to converge to the ground state as time goes to infinity. Under an infrared regularization (or provided that some gauge invariance is present in the model, as in the case of the standard model of non-relativistic QED), a ground state exists (i.e. the bottom of the spectrum is an eigenvalue), see [BFS3,Ge1,GLL]. On the other hand, if the model is "infrared singular", the absence of a ground state is established in different papers, see e.g. [LMS,DG3,CF,HH,Pa,AFGG]. In this case one can sometimes establish the existence of a ground state in a non-Fock representation (see for example [Ar]).

Spectral theory

In the massless case, the spectrum of H_g is equal to its essential spectrum, given by the semi-axis $[\inf \sigma(H_g), \infty)$, whereas in the massive case, the essential spectrum of H_g is given by the semi-axis $[\inf \sigma(H_g) + m, \infty)$ (see [DG1]). Generally speaking, Mourre's theory is a very powerful method to study essential spectra. In QFT, many authors have considered (different versions of) Mourre's theory in this respect. For small coupling, Mourre's theory can be implemented with the generator of dilatations in Fock space as a conjugate operator (see [BFS1,FGS,CFFS,ABFG]). It shows in particular that the spectrum of H_g is purely absolutely continuous just above the ground state energy as expected from the physics (for massless models).

For arbitrary values of g, the generator of radial translations has turned out to be a more suitable choice of conjugate operator (see [Sk,GGM]), requiring however the development of extensions of the usual Mourre theory. For a large class of QFT models, the method gives absence of singular continuous spectrum and local finiteness of point spectrum (the drawback being that, up to now, some infrared regularization combined with a confining assumption on the electronic part of the Hamiltonian are necessary to implement the theory). Besides, this extension of Mourre's theory allows one to study instability of embedded eigenvalues according to the Fermi Golden Rule criterion (see [DJ,Go,FMS]).

Let us observe that the electronic part of the model may exhibit a finite ionization threshold (this is the case, for instance, if H_{part} is a Schrödinger operator with Coulomb-type interaction). As far as we know, the spectrum of H_g above the ionization threshold (and near the threshold) has never been precisely studied. We will devote a part of the present project to this question.

Resonances
Through the theory of resonances, the disappearance of unperturbed eigenvalues according to Fermi Golden Rule can be described in a more precise way than by using Mourre's theory. According to the Aguilar-Balslev-Combes theory, resonances are defined as complex eigenvalues of a non self-adjoint family of operators $H_g(\theta)$ obtained from H_g by complex dilatations. In [BFS], Bach, Fröhlich and Sigal have developed a rigorous spectral renormalization group in order to prove the existence of resonances for H_g. Extensions of the method were later given in [Fa,Si] in order to cover more complicated models.

As shown in [BFS3,AFFS], resonances are associated to metastable states in the sense that any initial state of energy close to a given resonance will exhibit exponential decay, the survival probability of the considered state being close to the inverse of the imaginary part of the resonance. Results established in [AFFS] however hold only up to time of order $O(g^{-2})$; the decay of resonant states for large time remains an open problem. This question is in fact strongly related to the determination of the singularity of the resolvent of H_g near resonances, which is a very important problem, presently poorly understood.

This is one of the goals of this project to carry out a more precise study of the resolvent in a neighborhood of resonances.

Scattering theory
The first step in the construction of scattering theory for QFT Hamiltonians is the proof of the existence of so-called asymptotic creation and annihilation operators. For $t \to +\infty$, the Hilbert space $\mathcal{K}^+$ of asymptotic matter is defined as the space of states that contain no asymptotically free bosons, in other words, $\mathcal{K}^+$ is the space of states that are annihilated by all asymptotic annihilation operators. The full asymptotic Hilbert space $\mathcal{H}^+$ is the tensor

product of $\mathcal{K}^+$ by the Fock space of asymptotically free bosons, and the wave operator is an isometric operator from $\mathcal{H}^+$ to $\mathcal{H}$, which intertwines the usual and asymptotic field operators. As shown in [HK1], wave operators are unitary for massive models; however they may not be unitary in the massless case.

Physically, asymptotic completeness means that, for large time, states evolve according to a simpler evolution representing bound states together with free bosons that escape to infinity. Introducing the same definitions for $t \to -\infty$ as those which were given in the case $t \to +\infty$, asymptotic completeness is the property that

$$\mathcal{K}^+ = \mathcal{K}^- = \mathrm{Ran}(1_{\mathrm{pp}}(H_g)).$$

In [DG1], asymptotic completeness is established for a class of massive QFT models. The proof uses an intermediate result, called geometric asymptotic completeness, which roughly speaking shows that $\mathcal{K}^+$ is equal to the states that do not propagate faster than $o(t)$.

In [Ge2,DG3], scattering theory for massless models is addressed. In [Ge2], the model is infrared regular, and geometric asymptotic completeness is established. In [DG3], the model is infrared singular, so that scattering theory has to be modified accordingly; it is shown that the CCR representations obtained from the asymptotic fields contain non-Fock, coherent representations.

We observe that in [DG1,Ge2,DG3], the electronic part of the Hamiltonian is supposed to be confined. For small coupling, a slightly different approach that does not require this confining assumption is given in [FGSchl]. In the latter paper an infrared cutoff is imposed in the interaction.

Very recently, important breakthroughs were made towards a proof of asymptotic completeness for massless models. In [BoFS], a maximal velocity estimate for photons in non-relativistic QED has been established. It states that, asymptotically as time tends to infinity, the probability to find photons propagating faster than the speed of light vanishes. In [DK], for the spin-boson model, it is proven that the number of emitted particles remains bounded, uniformly in time.

One of the main objectives of this project is to show asymptotic completeness for massless Pauli-Fierz models.

QFT in curved space-time

QFT in curved space-time is an extension of the original QFT defined on Minkowski space-time. If the metric on the space-time is static, it is possible to introduce rigorous interacting models and study their spectral properties. This was first done in [GHPS1,GHPS2,GHPS3] where the Nelson model on a static space-time is considered and the question of the existence of a ground state is addressed. This model is obtained by the quantization of a non-relativistic particle linearly coupled to a Klein-Gordon field on a static space-time. The introduction of the metrics leads to an effective positive mass that depends on the position and may decay to zero at infinity.

It is shown that if the mass of the bosons decays sufficiently fast at infinity, the model does not have a ground state, whereas if the decay is not too strong, a ground state exists. In some sense, this can be seen as a reminiscence of the dichotomy observed between the massless and the massive Nelson model in the Minkowski space-time.

Nevertheless, the proofs presented in [GHPS1,GHPS2,GHPS3] are not straightforward adaptations of the corresponding proofs for the usual Nelson model. In particular several ingredient in the proofs are borrowed from pseudo-differential analysis and functional integration methods.

The works [GHPS1,GHPS2,GHPS3] give a satisfactory and fairly complete answer to the question of the existence of a ground state for the Nelson model in a static space-time. Spectral analysis above the ground state energy and scattering theory yet remain to be studied; part of our project will be dedicated to this issue. Moreover, the class of metrics considered in [GHPS1,GHPS2,GHPS3] does not include interesting physical examples like Schwarzschild or de Sitter space-time, and this is certainly a direction to follow.

Weak interaction and relativistic QED

Non-relativistic QED does not allow for a study of phenomena involving charged particles with very large velocities. Nevertheless, there exist several different ways to define in a mathematically rigorous way a model describing relativistic charged particles coupled to the quantized radiation field. A naïve approach is to replace the Schrödinger operator of non-relativistic QED by the Dirac operator. However, this does not define a semi-bounded operator, and hence, for instance, the question of the existence of a ground state does not make sense. A possibility is then to restrict the whole system to the positive spectral subspace of the free Dirac operator, the resulting operator being the so-called Brown-Ravenhall Hamiltonian. Other closely related points of view are given by the so-called no-pair Hamiltonian and the semi-relativistic Pauli-Fierz operator. The existence of a ground state for these models, under suitable assumptions, has been established recently in [KMS1,KMS2].

In this project, we will consider another approach, based on second quantization (and normal ordering) of Dirac-Coulomb fields. The model obtained by this construction takes into account the creation and annihilation of pairs of particle/anti-particle, namely one electron and one positron. Electrons and positrons are represented by vectors in anti-symmetric Fock spaces, and the total Hamiltonian acts in the tensor product of the Fock spaces for the photons, the electrons and the positrons respectively. We emphasize that the unboundedness of the number of electrons (and of positrons) gives rise to subtle questions that are not present in the previously cited models. In fact the mathematical analysis of this second quantized version of relativistic QED can be expected to be more difficult than the corresponding ones for the no-pair Hamiltonian and the semi-relativistic Pauli-Fierz operator.

In [BDG], such a second quantized relativistic QED model with ultraviolet and infrared cutoffs is considered. It is proved that the Hamiltonian is self-adjoint and has a ground state for sufficiently small values of the coupling constant. In this project, we will pursue the spectral analysis of this Hamiltonian, avoiding in addition the infrared cutoff assumption. Physically, since the mass of photons is equal to 0, there is naturally no infrared cutoff in relevant models of QED.

Although the weak and the electromagnetic interactions are very different kinds of interactions in the physical point of view, it turns out that they provide fairly close mathematical models of QFT. In [BaG,ABFG], a mathematical model of the weak interaction describing the decay of the intermediate vector bosons W+/- is considered. It is proved that a ground state exists and, without requiring any infrared regularization, a limiting absorption principle is established just above the ground state energy.

In relation with the second quantized relativistic QED model mentioned above, we will also consider in this project spectral theory for mathematical models of the weak interaction.

2.3. OBJECTIVES, ORIGINALITY AND NOVELTY OF THE PROJECT

Generally speaking, the goal of this project is the study of spectral and scattering properties of QFT models. This is a very active thematic of research in mathematical physics that has never been the topic of an ANR project before.

Even in the case of the largely studied Pauli-Fierz Hamiltonians in Minkowski space-time related to non-relativistic QED, several questions remain unsolved. Two of our main objectives, namely the proof of asymptotic completeness for massless models and the study of the resolvent near resonances, are well-known important open problems. In both cases, the central difficulty lies into the fact that the bosons under consideration are massless, leading to the famous infrared problem: a state of arbitrary low energy can be composed of an arbitrary high number of soft bosons. Various techniques have been introduced in the recent years to control the number of such bosons but so far, the two aforementioned questions have remained beyond reach.

Our second main direction of work concerns the mathematical analysis of QFT models associated to physical situations that cannot be described by non-relativistic QED in Minkowski space-time. In particular, important physical phenomena, such that the Hawking effect or the Unruh effect, find their justification through QFT in curved space-time. For interacting models, a very first step in this direction is the study of Nelson-type models on a static space-time. As shown in [GHPS1,GHPS2,GHPS3], functional integration methods and pseudo-differential analysis can be used to answer the question of the existence of a ground state, but other spectral and scattering properties of the model remain to be studied

Likewise, to properly describe phenomena involving relativistic charged particles, one has to take into account the possible creation or annihilation of pairs of particle-antiparticle, as is done, for instance, in [BDG]. Mathematically, the fact that the number of charged particles is not conserved leads to substantial technical difficulties, in particular, the spectral and scattering analyses of the model near thresholds is a very subtle problem. Up to now, only a few basic properties have been derived for such models, namely self-adjointness and existence of a ground state under an infrared regularization. Again, spectral and scattering analyses remain to be studied in this context.

To reach the objectives of this project, we shall be in contact, in particular, with the following external collaborators: L. Amour, C. Gérard, J. Fröhlich, J-C. Guillot, F. Hiroshima, J.S. Møller, I.M. Sigal, A. Suzuki.

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