TD4

ex1.

① $\begin{cases} 3x+5y=11 \\ 2x+3y=7 \end{cases} \iff \begin{cases} x+2y=4 \\ 2x+3y=7 \end{cases}$

$\iff \begin{cases} x+2y=4 \\ -y=-1 \end{cases} \iff \begin{cases} x=2. \\ y=1 \end{cases}$

② $\begin{cases} 2x+5y=10 \\ 2x+3y=8 \end{cases} \iff \begin{cases} 2x+5y=10 \\ -2y=-2 \end{cases} \iff \begin{cases} x=\frac{5}{2} \\ y=1 \end{cases}$

③ $\begin{cases} 6x+12y=30 \\ 3x+3y=9 \end{cases} \iff \begin{cases} x+2y=5 \\ x+y=3 \end{cases} \iff \begin{cases} x+2y=5 \\ 0-y=-2 \end{cases}$

$\iff \begin{cases} x=1 \\ y=2 \end{cases}$

ex2

notons x = le nombre de lapins

y = le nombre de poulets.

Alors $\begin{cases} x+y=27 \\ 4x+2y=72. \end{cases}$

$\iff \begin{cases} x+2y=27 \\ -2y=72-27\times 4=-36 \end{cases}$

$\iff \begin{cases} x=9 \\ y=18 \end{cases}$ donc il y a 9 lapins et 18 poulets.

exs

①
$$\begin{cases} -5x - y + 2z = -20 \\ -2x + 6y + 2z = 2. \\ 4x + 2y - 8z = -2 \end{cases}$$

$$\Leftrightarrow \begin{cases} -5x - y + 2z = -20 & L_1 \to L_1 \\ x - 3y - z = -1 & -\frac{1}{2}L_2 \to L_2 \\ 2x + y - 4z = -1 & \frac{1}{2}L_3 \to L_3 \end{cases}$$

$$\Leftrightarrow \begin{cases} x - 3y - z = -1 & L_2 \to L_1 \\ -5x - y + 2z = -20 & L_1 \to L_2 \\ 2x + y - 4z = -1 & L_3 \to L_3 \end{cases}$$

$$\Leftrightarrow \begin{cases} x - 3y - z = -1 & L_1 \to L_1 \\ -16y - 3z = -25 & 5 \times L_1 + L_2 \to L_2 \\ 7y - 2z. = 1 & L_3 - 2L_1 \to L_3 \end{cases}$$

$$\Leftrightarrow \begin{cases} x - 3y - z = -1 & L_1 \to L_1 \\ y + \frac{3}{16}z = \frac{25}{16} & -\frac{1}{16}L_2 \to L_2 \\ 7y - 2z = 1 & L_3 \to L_3 \end{cases}$$

$$\Leftrightarrow \begin{cases} x - 3y - z = -1 & L_1 \to L_1 \\ y + \frac{3}{16}z = \frac{25}{16} & L_2 \to L_2 \\ (-2 - 7 \times \frac{3}{16})z = 1 - 7 \times \frac{25}{16} & -7L_2 + L_3 \to L_3 \end{cases}$$

$$\Leftrightarrow \begin{cases} x - 3y - z = -1. & L_1 \to L_2 \\ y + \frac{3}{16}z = \frac{25}{16} & L_2 \to L_2 \\ -53z = -159 & 16L_3 \to L_3 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = 5 \\ y = 1 \\ z = 3 \end{cases}$$

②. $$\begin{cases} -9x + 9y + 6z = 114 \\ 4x - 7z = -91 \\ -x - 2z = -26 \end{cases} \qquad (*)$$

on résout d'abord les deux dernières équations

$$\begin{cases} 4x - 7z = -91 \\ -x - 2z = -26 \end{cases}$$

$$\Leftrightarrow \begin{cases} x + 2z = 26 \\ 4x - 7z = -91 \end{cases} \Leftrightarrow \begin{cases} x + 2z = 26 \\ -15z = -195 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = 0 \\ z = 13. \end{cases}$$

donc la première équation de $(*)$ nous donne

$$-9 \times 0 + 9 \times y + 6 \times 13 = 114.$$

$$\Rightarrow \quad 9 \times y = 114 - 6 \times 13 = 114 - 78 = 36$$

donc $y = 4$

$$\Rightarrow \begin{cases} x = 0 \\ y = 4 \\ z = 13 \end{cases}$$

ex4

① $\begin{cases} x - 3y + 2z = 8 \\ -x + 3y - 4z = -16 \end{cases}$

$\Leftrightarrow \begin{cases} x - 3y + 2z = 8 \\ -2z = -8 \end{cases}$

donc $\begin{cases} x = 3t \\ y = t \in \mathbb{R} \\ z = 4 \end{cases}$

② $\begin{cases} 6x + 3y + 1 = 10 \\ 6x + 3y + 3 = 12. \end{cases}$

$\Leftrightarrow \begin{cases} 6x + 3y = 9 \\ 6x + 3y = 9 \end{cases}$

$\Leftrightarrow \begin{cases} 6x + 3y = 9 \\ 0 = 0 \end{cases}$

donc une infinité de solutions

$\begin{cases} x = \tilde{t} \in \mathbb{R} \\ y = 3 - 2\tilde{t}. \end{cases}$

ex 5

1

$$\begin{cases} x + 2y = t^2 \\ 4x + 3y = 1 \end{cases}$$

$$\Leftrightarrow \begin{cases} x + 2y = t^2 \\ -5y = 1-4t^2 \end{cases} \Leftrightarrow \begin{cases} x = t^2 - \frac{8t^2-2}{5} = \frac{-3t^2+2}{5} \\ y = \frac{4t^2-1}{5}. \end{cases}$$

2

$$\begin{cases} 2tx + 9y = \cancel{27}\ 21 \\ 8x + ty = 14 \end{cases}$$

$$\Leftrightarrow \begin{cases} 8x + ty = 14 \\ 2tx + 9y = 21 \end{cases} \Leftrightarrow \begin{cases} x + \frac{t}{8}y = \frac{7}{4} \\ 2tx + 9y = 21 \end{cases}$$

$$\Leftrightarrow \begin{cases} x + \frac{t}{8}y = \frac{7}{4} \\ (9 - \frac{t^2}{4})y = 21 - \frac{7t}{2} \end{cases}$$

donc : * si $9 - \frac{t^2}{4} \neq 0$ (i.e. $t \neq \pm 6$)

$\exists$ unique solution
$$\begin{cases} x = \cdots \\ y = \dfrac{21 - \frac{7t}{2}}{9 - \frac{t^2}{4}} = \dfrac{84-14t}{36-t^2}. \end{cases}$$

* si $9 - \frac{t^2}{4} = 0$ et $21 - \frac{7t}{2} = 0$ (c'est-à-dire : $t = 6$)

$\exists$ une infinité de solutions
$$\begin{cases} x = \frac{7}{4} - \frac{3}{4}t \\ y = t \in \mathbb{R} \end{cases}$$

* Si $9-\frac{t^2}{4}=0$ et $21-\frac{7\times t}{2}\neq 0$, c'est-à-dire, si $t=-6$

il n'y a pas de solution car on a dans le cas là

$$0 = 0\cdot y = 21-\frac{7\times(-6)}{2} = 42 \neq 0 \ !$$

(3)

$$\begin{cases} 2x - (t-1)y = 4 \\ (t+2)x + (2t+1)y = t-1 \end{cases} \Leftrightarrow \begin{cases} x - \frac{t-1}{2}y = 2 \\ (t+2)x + (2t+1)y = t-1 \end{cases}$$

$$\Leftrightarrow \begin{cases} x - \frac{t-1}{2}y = 2 \\ \left(2t+1+\frac{(t-1)(t+2)}{2}\right)y = t-1-2(t+2) \end{cases}$$

$$\Leftrightarrow \begin{cases} x - \frac{t-1}{2}y = 2 \\ (t^2+5t)y = -2t-10 \end{cases}$$

* Si $t^2+5t\neq 0 \Rightarrow$ $y = \frac{-2t-10}{t^2+5t} = \frac{-2}{t}$

donc $x = 2+\frac{t-1}{2}y = 2-\frac{t-1}{t} = \frac{t+1}{t}$

$$\Rightarrow \begin{cases} x = \frac{t+1}{t} \\ y = -\frac{2}{t} \end{cases} \quad (t\neq 0,\ t\neq -5)$$

* Si $t^2+5t=0$ et $-2t-10=0 \Leftrightarrow t=-5$

$\Rightarrow \exists$ une infinité de solutions.

$$\begin{cases} x = 2+\frac{t-1}{2}s = 2-3s \\ y = s \in \mathbb{R} \end{cases}$$

$*$ Si $t^2+5t=0$ et $-2t-10\neq 0 \iff t=0$

pas de solution

4 $\begin{cases} 3x+y-z=3 \\ 2x-3y+2z=3 \\ x+4y+tz=0. \end{cases} \iff \begin{cases} x+4y-3z=0 \\ 2x-3y+2z=3 \\ x+4y+tz=0. \end{cases}$

$\iff \begin{cases} x+4y-3z=0 \\ \quad -11y+8z=3 \\ \quad\quad (t+3)z=0 \end{cases}$

Donc $*$ Si $t+3\neq 0 \iff t\neq -3$

le système admet une unique solution $\begin{cases} x=\frac{4\times 3}{11}=\frac{12}{11} \\ y=-\frac{3}{11} \\ z=0. \end{cases}$

$*$ Si $t+3=0$ i.e. $t=-3$

$\rightsquigarrow$ $\exists$ une infinité de solutions

$\begin{cases} x=3s-4y=3s-\frac{32s-12}{11}=\frac{s+12}{11} \\ y=(8s-3)/11 \\ z=s\in\mathbb{R} \end{cases}$

⑤. $\begin{cases} 7x - 3y + tz = 29 \\ 70x + 2y + 5z = t. \\ 19x + y + 16z = 41 \end{cases}$

$\Leftrightarrow \begin{cases} 7x - 3y + tz = 29 \\ 32y + (5-10t)z = t - 290 \\ (1 + 3\times\frac{19}{7})y + (16 - t\times\frac{19}{7})z = 41 - 29\times\frac{19}{7} \end{cases}$

$\Leftrightarrow \begin{cases} 7x - 3y + tz = 29 \\ y + \frac{5-10t}{32} z = \frac{t-290}{32} \\ +\frac{64}{7} y + \frac{112-19t}{7} z = -\frac{264}{7} \end{cases}$

$\Leftrightarrow \begin{cases} 7x - 3y + tz = 29 \\ 32y + (5-10t)z = t - 290 \\ 64y + (112-19t)z = -264. \end{cases}$

$\Leftrightarrow \begin{cases} 7x - 3y + tz = 29 \\ 32y + (5-10t)z = t - 290 \\ (102+t)z = 316-2t. \end{cases}$

Donc * Si $t \neq -102$ $\Rightarrow$ unique solution $\begin{cases} x = \ldots \\ y = \ldots \\ z = \frac{316-2t}{102+t}. \end{cases}$

* Si $t = -102$, la dernière équation nous donne

$$0 = 0 \cdot z = 316 - 2\times(-102) \neq 0$$

$\Rightarrow$ pas de solution

6

$$\begin{cases} x + y = 2t \\ -x + 2y + z = 4 \\ 4x + y - z = 2 \end{cases}$$

$$\Longleftrightarrow \begin{cases} x + y = 2t \\ 3y + z = 4 + 2t. \\ -3y - z = 2 - 8t. \end{cases}$$

$$\Longleftrightarrow \begin{cases} x + y = 2t \\ 3y + z = 4 + 2t \\ 0 = 6 - 6t \end{cases}$$

<u>Donc</u> : * Si $6-6t \neq 0 \Rightarrow$ pas de solution

* Si $6-6t = 0$ i.e. $t=1 \Rightarrow \exists$ une infinité de solutions

$$\begin{cases} x = 2 - s \\ y = s \in \mathbb{R} \\ z = 6 - 3s \end{cases}$$

ex6

① $\begin{cases} 2x + 3y = 4 \\ 3x + 7y = 0 \\ 4x + y = 1 \end{cases}$

$\Leftrightarrow \begin{cases} 2x + 3y = 4 \\ \frac{5}{2}y = -6 \\ -5y = -7. \end{cases} \Leftrightarrow \begin{cases} 2x + 3y = 4 \\ 5y = -12 \\ 0 = -19 \end{cases}$

Donc pas de solution

② $\begin{cases} 2x + 3y = 4 \\ 3x + 7y = 0 \\ 4x + y = 20 \end{cases} \Leftrightarrow \begin{cases} 2x + 3y = 4 \\ \frac{5}{2}y = -6 \\ -5y = 12. \end{cases}$

$\Leftrightarrow \begin{cases} 2x + 3y = 4 \\ 5y = -12 \\ 0 = 0 \end{cases}$

$\Leftrightarrow \begin{cases} x = \frac{1}{2}\left(4 - 3\times\frac{-12}{5}\right) = \cdots \\ y = \frac{-12}{5} \end{cases}$

3

$$\begin{cases} 2x + 3y = 4 \\ 3x + 7y = 0 \\ 4x + 3y = 20 \end{cases}$$

$$\Leftrightarrow \begin{cases} 2x + 3y = 4 \\ \frac{5}{2} y = -6 \\ -3y = +12 . \end{cases} \Leftrightarrow \begin{cases} 2x + 3y = 4 \\ 5y = -12 \\ y = -4 \end{cases}$$

$\rightsquigarrow$ pas de solution

4

$$\begin{cases} 2x + 3y = 0 \\ 3x + 7y = 0 \\ 4x + 3y = 0 . \end{cases}$$

$$\Leftrightarrow \begin{cases} 2x + 3y = 0 . \\ \frac{5}{2} y = 0 . \\ -3y = 0 . \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ y = 0 . \end{cases}$$