Optimal rotary control of the cylinder wake using POD reduced order model

Michel Bergmann, Laurent Cordier & Jean-Pierre Brancher

Michel.Bergmann@ensem.inpl-nancy.fr

Laboratoire d'Énergétique et de Mécanique Théorique et Appliquée

UMR 7563 (CNRS - INPL - UHP)

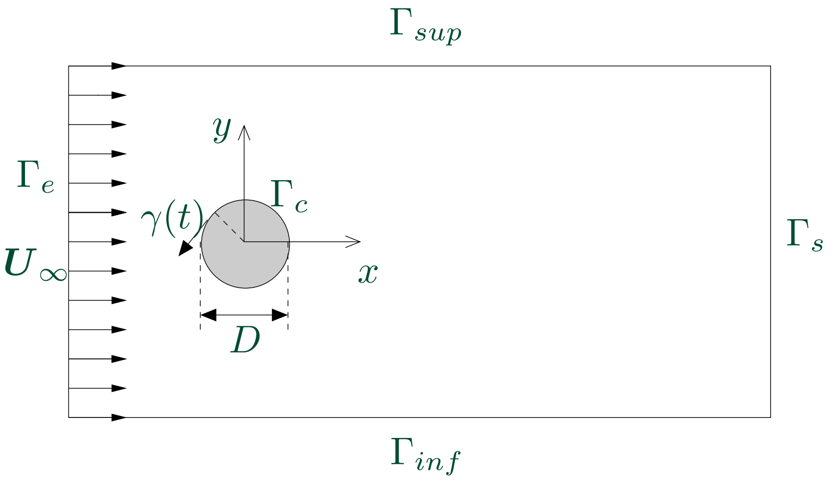
ENSEM - 2, avenue de la Forêt de Haye

BP 160 - 54504 Vandoeuvre Cedex, France

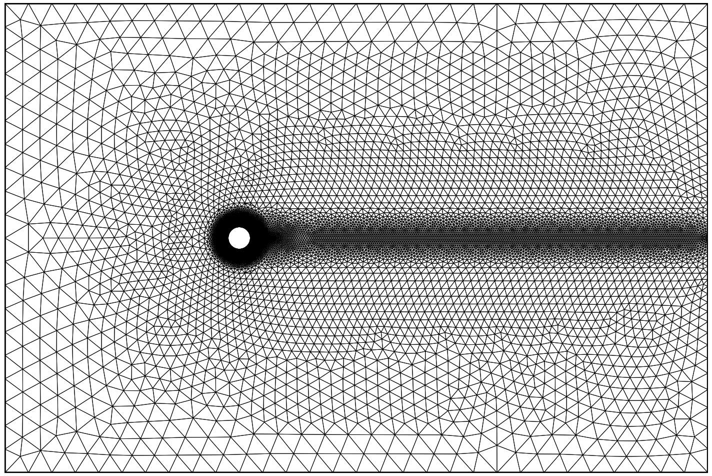
Outline

I - Configuration and numerical method

- Two dimensional flow around a circular cylinder at $R_e = 200$
- Viscous, incompressible and Newtonian fluid
- Cylinder oscillation with a tangential velocity $\gamma(t)$

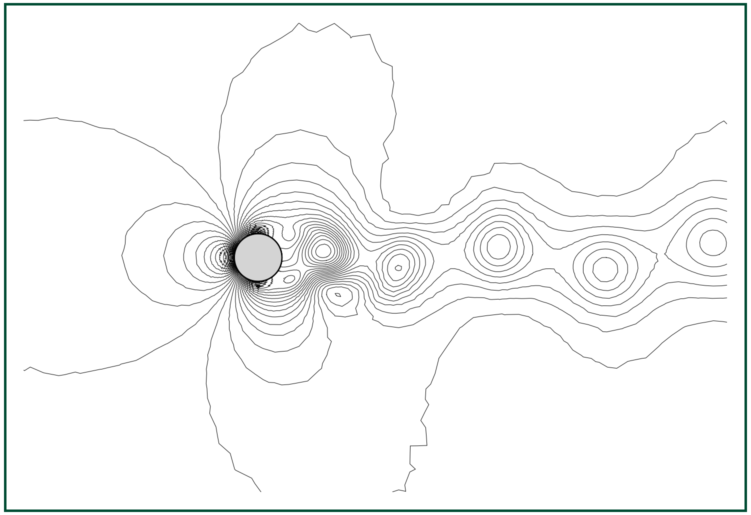


- Fractional steps method in time
- Finite Elements Method (FEM) in space

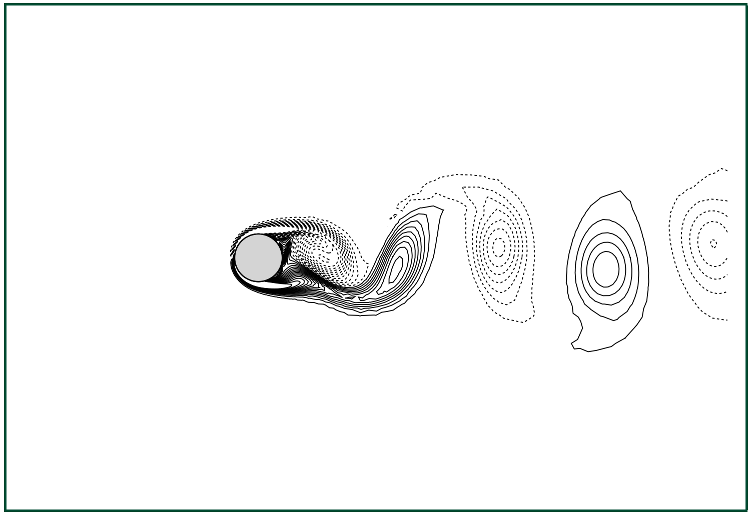


► *Numerical code written by M.Braza (IMFT-EMT2) & D.Ruiz (ENSEEIHT)*

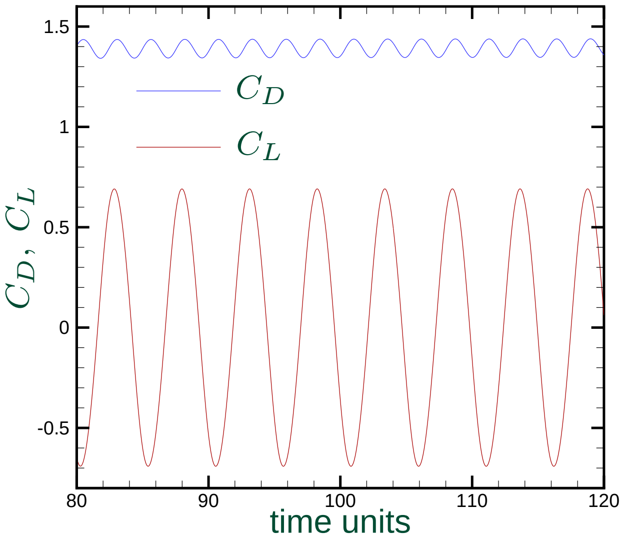
I - Configuration and numerical method



Iso pressure at $t = 100$.



Iso vorticity at $t = 100$.



Aerodynamics coefficients.

Authors	S_t	C_D
Braza *et al.* (1986)	0.2000	1.4000
Henderson *et al.* (1997)	0.1971	1.3412
He *et al.* (2000)	0.1978	1.3560
this study	**0.1983**	**1.3972**

Strouhal number and drag coefficient.

II - Optimal control *Definition*

Mathematical method allowing to determine without empiricism a control law starting from the optimization of a cost functional.

- State equation $\mathcal{F}(\phi, c) = 0$;
  (Navier-Stokes + C.I. + C.L.)

- Control variables c ;
  (Blowing/suction, design parameters ...)

- Cost functional $\mathcal{J}(\phi, c)$.
  (Drag, lift, ...)

Find a control law c and state variable ϕ such that the cost functional $\mathcal{J}(\phi, c)$ reach an extremum under the constrain $\mathcal{F}(\phi, c) = 0$.

II - Optimal control *Lagrange multipliers*

Constrained optimization $\Rightarrow$ unconstrained optimization

- Introduction of Lagrange multipliers ξ.

- Lagrange functional :

$$\mathcal{L}(\phi, c, \xi) = \mathcal{J}(\phi, c) - < \mathcal{F}(\phi, c), \xi >$$

- Force $\mathcal{L}$ to be stationary $\Rightarrow$ look for $\delta\mathcal{L} = 0$:

$$\delta\mathcal{L} = \frac{\partial \mathcal{L}}{\partial \phi}\delta\phi + \frac{\partial \mathcal{L}}{\partial c}\delta c + \frac{\partial \mathcal{L}}{\partial \xi}\delta\xi = 0$$

- Suppose ϕ, c and ξ independant each other :

$$\frac{\partial \mathcal{L}}{\partial \phi}\delta\phi = \frac{\partial \mathcal{L}}{\partial c}\delta c = \frac{\partial \mathcal{L}}{\partial \xi}\delta\xi = 0$$

II - Optimal control *optimal system*

- State equation $(\frac{\partial \mathcal{L}}{\partial \xi}\delta\xi = 0)$:

$$\mathcal{F}(\phi, c) = 0$$

- Co-sate equation $(\frac{\partial \mathcal{L}}{\partial \phi}\delta\phi = 0)$:

$$\left(\frac{\partial \mathcal{F}}{\partial \phi}\right)^* \xi = \left(\frac{\partial \mathcal{J}}{\partial \phi}\right)^*$$

- Optimality condition $(\frac{\partial \mathcal{L}}{\partial c}\delta c = 0)$:

$$\left(\frac{\partial \mathcal{J}}{\partial c}\right)^* = \left(\frac{\partial \mathcal{F}}{\partial c}\right)^* \xi$$

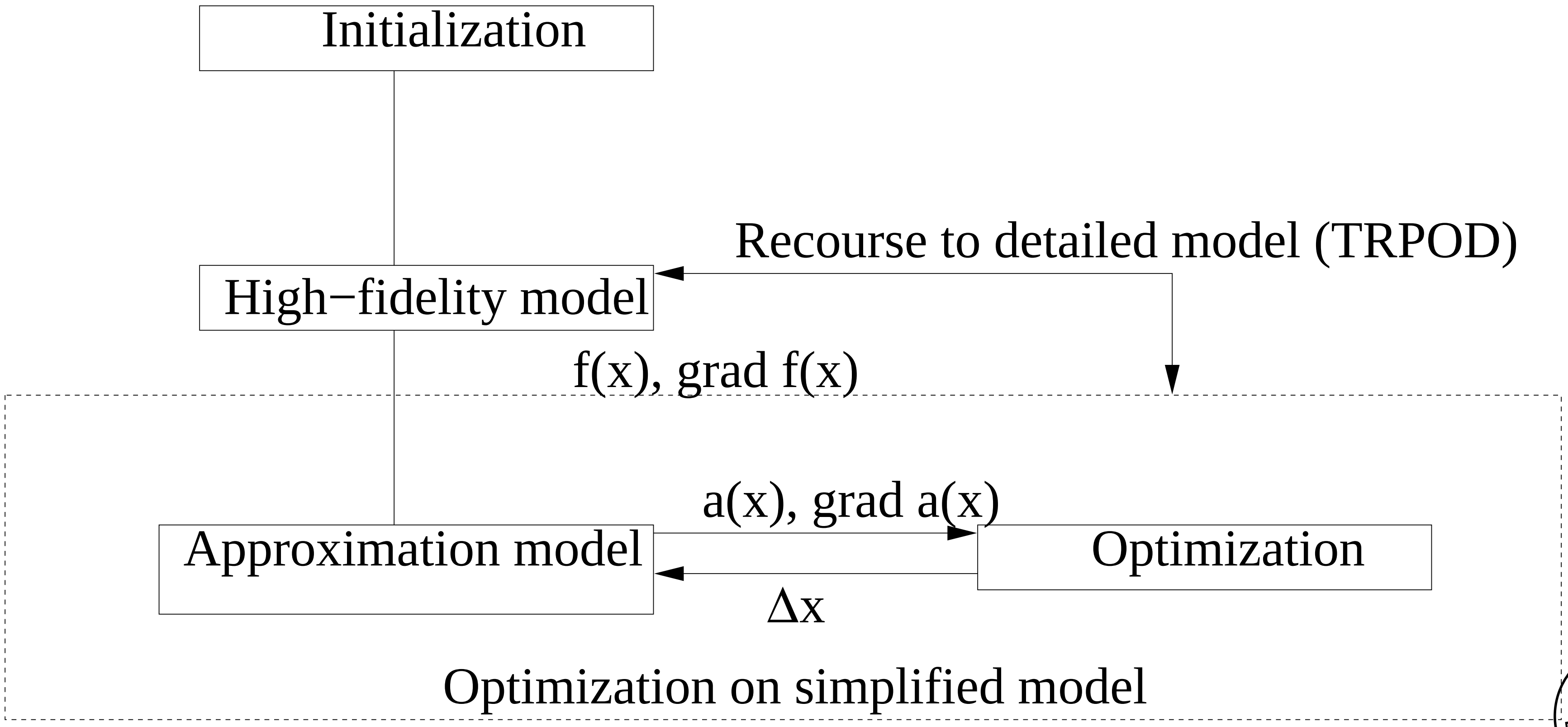
$\Rightarrow$ Expensive method in CPU time and storage memory for large system !

$\Rightarrow$ Ensure only a local *(generally not global)* minimum

II - Optimal control *Reduced Order Model (ROM)*

"without an inexpensive method for reducing the cost of flow computation, it is unlikely that the solution of optimization problems involving the three dimensional unsteady Navier-Stokes system will become routine"

M. Gunzburger, 2000



II - Proper Orthogonal Decomposition (POD)

- Introduced in fluid mechanics (turbulence context) by Lumley (1967).

- Look for a realization $\phi(\boldsymbol{X})$ which is closer, in an average sense, to the realizations $\boldsymbol{u}(\boldsymbol{X})$. ($\boldsymbol{X} = (\boldsymbol{x}, t) \in \mathcal{D} = \Omega \times \mathbb{R}^+$)

- $\boldsymbol{\phi}(\boldsymbol{X})$ solution of the problem : $\max\limits_{\boldsymbol{\phi}} \langle |(\boldsymbol{u}, \boldsymbol{\phi}|^2 \rangle$ s.t. $\|\boldsymbol{\phi}\|^2 = 1.$

- Snapshots method, Sirovich (1987) :

$$\int_T C(t, t') a^{(n)}(t')\, dt' = \lambda^{(n)} a^{(n)}(t).$$

- Optimal convergence L^2 *norm* (energy) of $\phi(\boldsymbol{X})$
  $\Rightarrow$ Dynamical order reduction is possible.

- Decomposition of the velocity field :

$$\boldsymbol{u}(\boldsymbol{x}, t) = \sum_{i=1}^{N_{POD}} a^{(i)}(t) \boldsymbol{\phi}^{(i)}(\boldsymbol{x}).$$

III - Reduced Order Model of the cylinder wake (ROM)

- Galerkin's projection of *NSE* on the POD basis :

$$\left(\boldsymbol{\phi}^{(i)},\, \frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u}\right) = \left(\boldsymbol{\phi}^{(i)},\, -\boldsymbol{\nabla} p + \frac{1}{Re}\Delta \boldsymbol{u}\right).$$

- Integration by parts (Green's formula) leads :

$$\left(\boldsymbol{\phi}^{(i)},\, \frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u}\right) = \left(p,\, \boldsymbol{\nabla}\cdot\boldsymbol{\phi}^{(i)}\right) - \frac{1}{Re}\left((\boldsymbol{\nabla}\otimes\boldsymbol{\phi}^{(i)})^T,\, \boldsymbol{\nabla}\otimes\boldsymbol{u}\right) - [p\,\boldsymbol{\phi}^{(i)}] + \frac{1}{Re}[(\boldsymbol{\nabla}\otimes\boldsymbol{u})\boldsymbol{\phi}^{(i)}].$$

with $[\boldsymbol{a}] = \int_\Gamma \boldsymbol{a}\cdot\boldsymbol{n}\, d\Gamma$ and $(A,\, B) = \int_\Omega A : B\, d\Omega = \sum_{i,\,j} \int_\Omega A_{ij} B_{ji}\, d\Omega$.

III - Reduced Order Model of the cylinder wake (ROM)

▶ Velocity decomposition with N_{POD} modes :

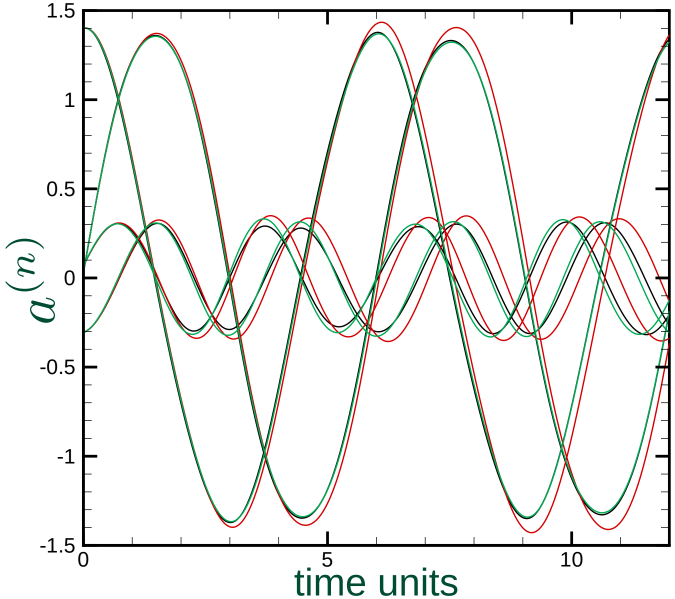
$$\boldsymbol{u}(\boldsymbol{x},t) = \boldsymbol{u}_m(\boldsymbol{x}) + \gamma(t)\,\boldsymbol{u}_c(\boldsymbol{x}) + \sum_{k=1}^{N_{POD}} a^{(k)}(t)\boldsymbol{\phi}^{(k)}(\boldsymbol{x}).$$

▶ Reduced order dynamical system where only N_{gal} $(\ll N_{POD})$ modes are retained (state's equation) :

$$\left\{\begin{aligned} \frac{d\,a^{(i)}(t)}{d\,t} =& \mathcal{A}_i + \sum_{j=1}^{N_{gal}} \mathcal{B}_{ij}\,a^{(j)}(t) + \sum_{j=1}^{N_{gal}}\sum_{k=1}^{N_{gal}} \mathcal{C}_{ijk}\,a^{(j)}(t)a^{(k)}(t) \\ & + \mathcal{D}_i\,\frac{d\,\gamma}{d\,t} + \left(\mathcal{E}_i + \sum_{j=1}^{N_{gal}} \mathcal{F}_{ij}\,a^{(j)}(t)\right)\gamma + \mathcal{G}_i\gamma^2 \\ a^{(i)}(0) =& (\boldsymbol{u}(\boldsymbol{x},\,0),\,\boldsymbol{\phi}^{(i)}(\boldsymbol{x})). \end{aligned}\right.$$

$\mathcal{A}_i$, $\mathcal{B}_{ij}$, $\mathcal{C}_{ijk}$, $\mathcal{D}_i$, $\mathcal{E}_i$, $\mathcal{F}_{ij}$ and $\mathcal{G}_i$ depend on $\boldsymbol{\phi}$, $\boldsymbol{u}_m$, $\boldsymbol{u}_c$ and Re.

Integration and (optimal) stabilization of the POD ROM for
$\gamma = A\sin(2\pi S_t t)$, $A = 2$ et $S_t = 0.5$.
POD reconstruction errors $\Rightarrow$ temporal modes amplification

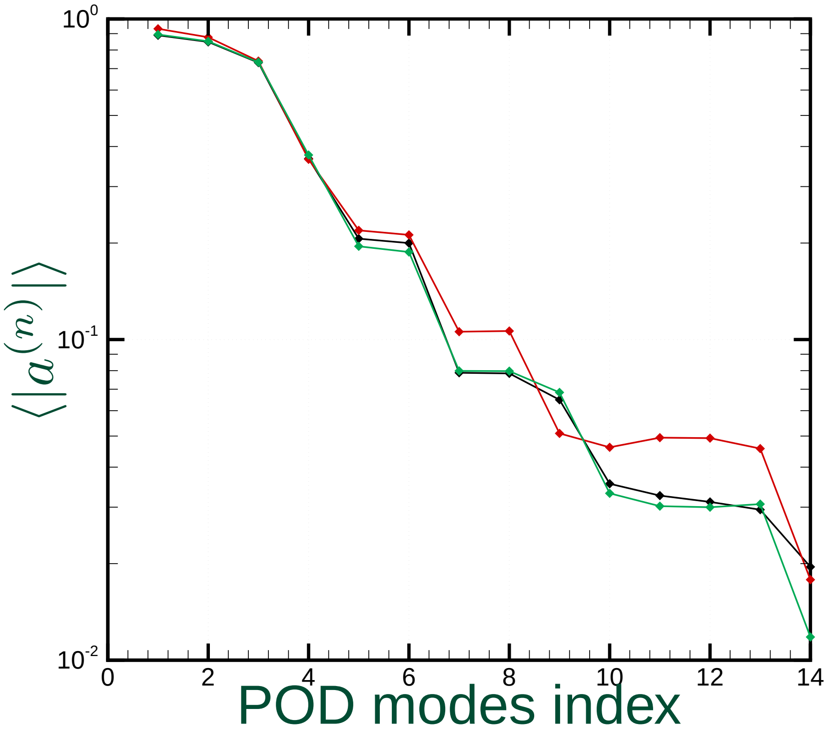


Temporal evolution of the first 6 POD temporal modes.

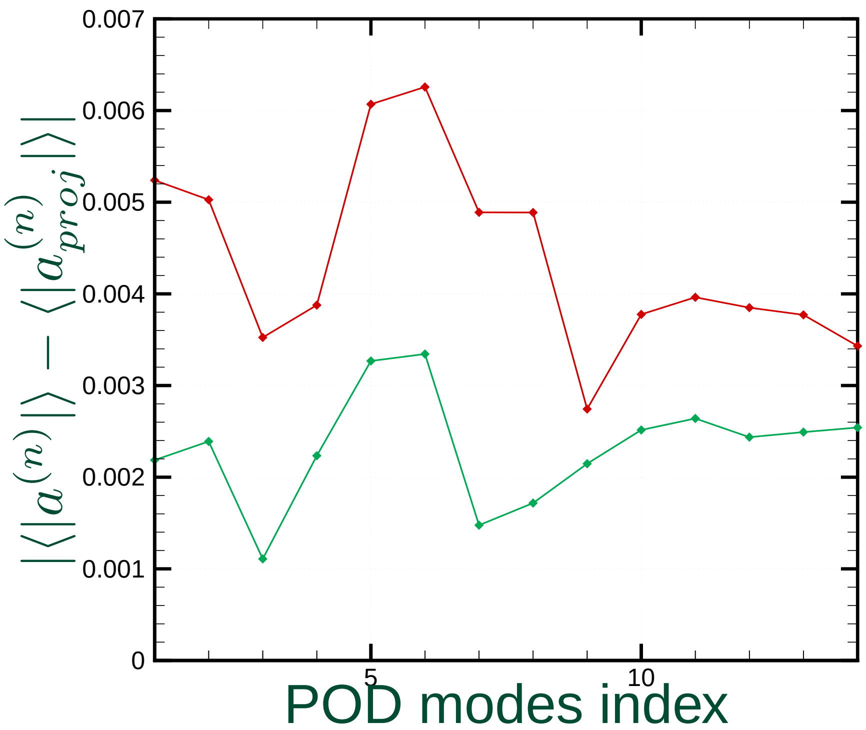
- **Causes :**
  - Extraction of large energetic eddies
  - Dissipation takes place in small eddies
- **Solution :**
  - Optimal addition of artificial viscosity on each POD mode

projection (Navier Stokes)
prediction before stabilisation (POD ROM)
prediction after stabilisation (POD ROM).

IV - Reduced Order Model of the cylinder wake *Stabilization*



Comparison of energetic spectrum.



Comparison of absolute errors.

► Good agreements between POD and DNS spectrum

► Reduced reconstruction error between predicted (POD) and projected (DNS) modes

$\Rightarrow$ Validation of the POD ROM

IV - Optimal control formulation based on reduced order model

- Objective functional :

$$\mathcal{J}(\boldsymbol{a}, \gamma(t)) = \int_0^T J(\boldsymbol{a}, \gamma(t))\, dt = \int_0^T \left(\sum_{i=1}^{N_{gal}} {a^{(i)}}^2 + \frac{\alpha}{2} \gamma(t)^2 \right) dt.$$

α : regularization parameter (penalization).

- Co-state's equations :

$$\begin{cases} \dfrac{d\,\xi^{(i)}(t)}{dt} = -\displaystyle\sum_{j=1}^{N_{gal}} \left(\mathcal{B}_{ji} + \gamma\, \mathcal{F}_{ji} + \sum_{k=1}^{N_{gal}} (\mathcal{C}_{jik} + \mathcal{C}_{jki})\, a^{(k)} \right) \xi^{(j)}(t) - 2a^{(i)} \\ \xi^{(i)}(T) = 0. \end{cases}$$

- Optimality condition (gradient) :

$$\delta\gamma(t) = -\sum_{i=1}^{N_{gal}} \mathcal{D}_i \frac{d\xi^{(i)}}{dt} + \sum_{i=1}^{N_{gal}} \left(\mathcal{E}_i + \sum_{j=1}^{N_{gal}} \mathcal{F}_{ij} a^{(j)} + 2\mathcal{G}_i \gamma(t) \right) \xi^{(i)} + \alpha\gamma.$$

IV - Optimal control formulation based on reduced order model

▶ $\gamma^{(0)}(t)$ done ; for $n = 0, 1, 2,$ ... and while a convergence criterium is not satisfied, do :

1. From $t = 0$ to $t = T$ solve the state's equations with $\gamma^{(n)}(t)$;
   $\hookrightarrow$ *state's variables* $a^{(n)}(t)$
2. From $t = T$ to $t = 0$ solve the co-state's equations with $a^{(n)}(t)$;
   $\hookrightarrow$ *co-state's variables* $\xi^{(n)}(t)$
3. Solve the optimality condition with $a^{(n)}(t)$ and $\xi^{(n)}(t)$;
   $\hookrightarrow$ *objective gradient* $\delta\gamma^{(n)}(t)$
4. New control law $\hookrightarrow \gamma^{(n+1)}(t) = \gamma^{(n)}(t) + \omega^{(n)}\,\delta\gamma^{(n)}(t)$

▶ End do.

V - Closed loop results *Generalities*

▶ No reactualization of the POD basis.

▶ The energetic representativity is *a priori* different to the dynamical one :

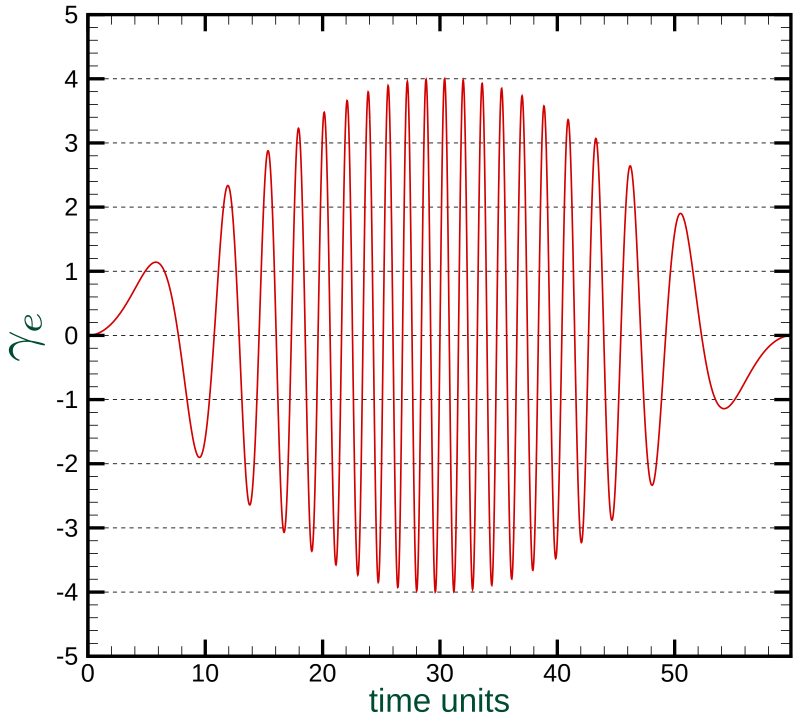
$\hookrightarrow$ possible inconvenient for control,

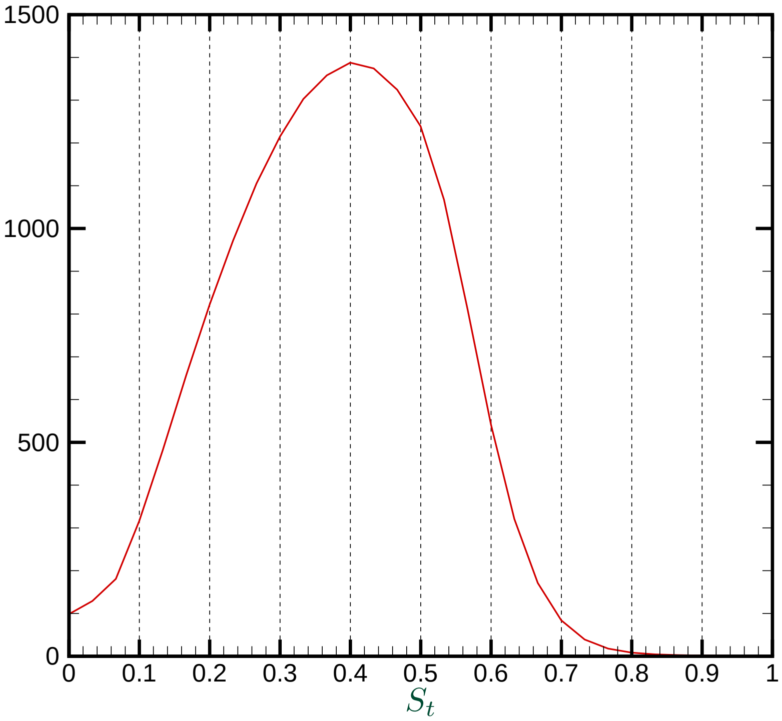
$\hookrightarrow$ a POD dynamical system represents *a priori* only the dynamics (and its vicinity) used to build the low dynamical model.

▶ Construction of a POD basis representative of a large range of dynamics :

$\hookrightarrow$*excitation of a great number of degrees of freedom scanning* $\gamma(t)$ *in amplitudes and frequencies.*

V - Closed loop results *Excitation*



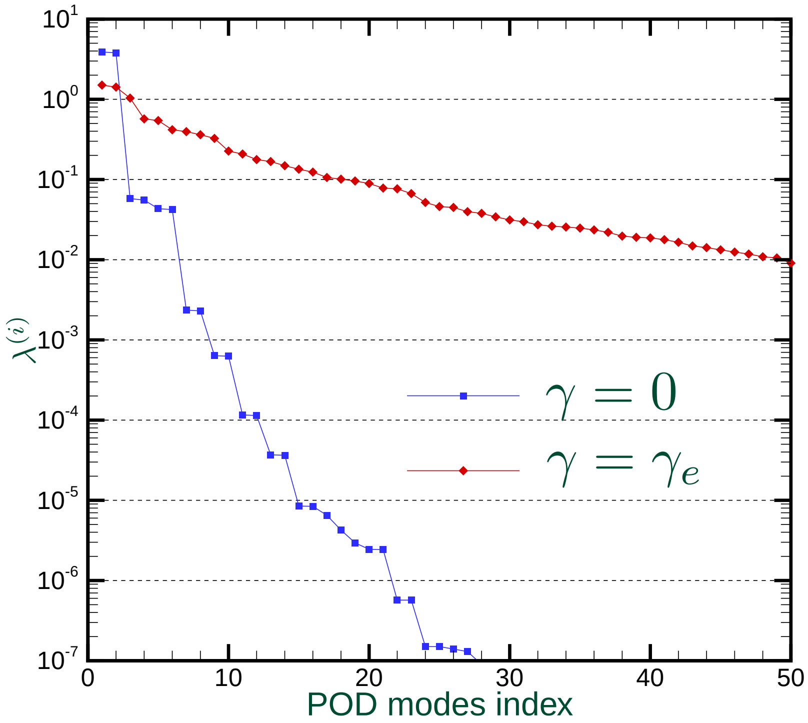


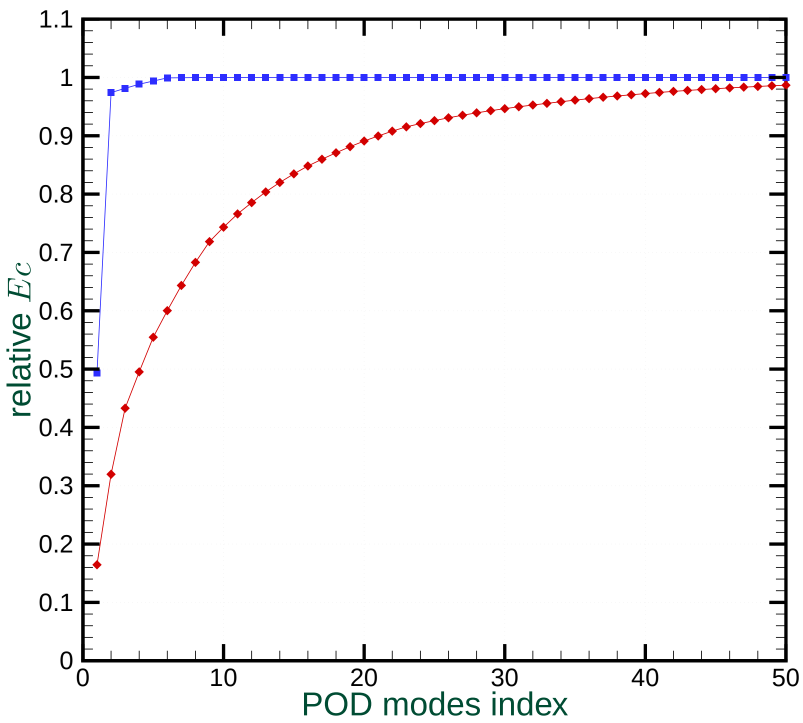
$$\gamma_e(t) = A_1 \sin(2\pi S_{t1}\, t) \times \sin(2\pi S_{t2}\, t - A_2 \sin(2\pi S_{t3}\, t))$$

with $A_1 = 4$, $A_2 = 18$, $S_{t1} = 1/120$, $S_{t2} = 1/3$ and $S_{t3} = 1/60$.

- $0 \leq$ amplitudes ≤ 4 and Fourier analysis $\Rightarrow 0 \leq$ frequencies ≤ 0.65
- γ_e initial control law in the iterative process.

V - Closed loop results *Energy*



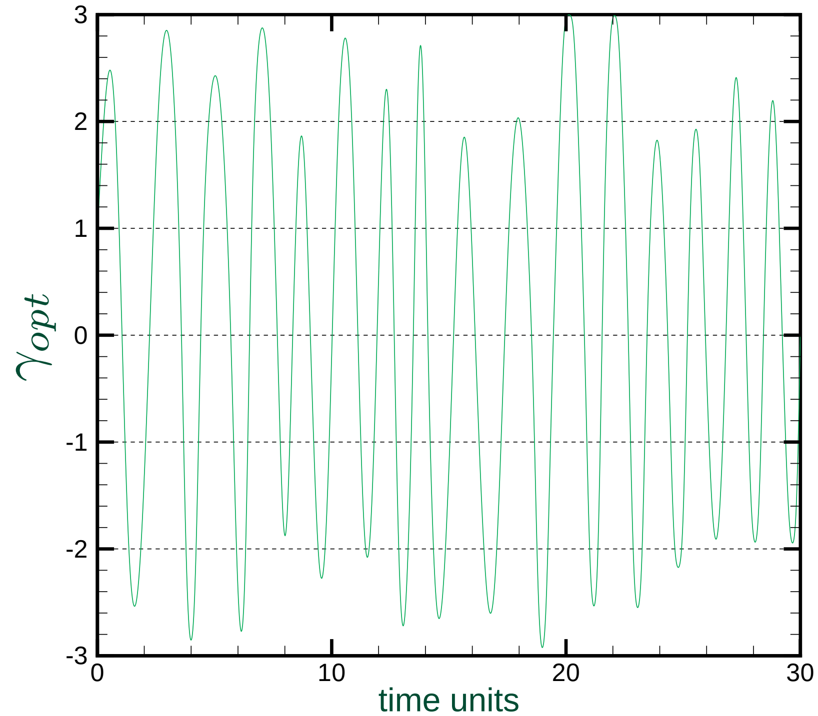


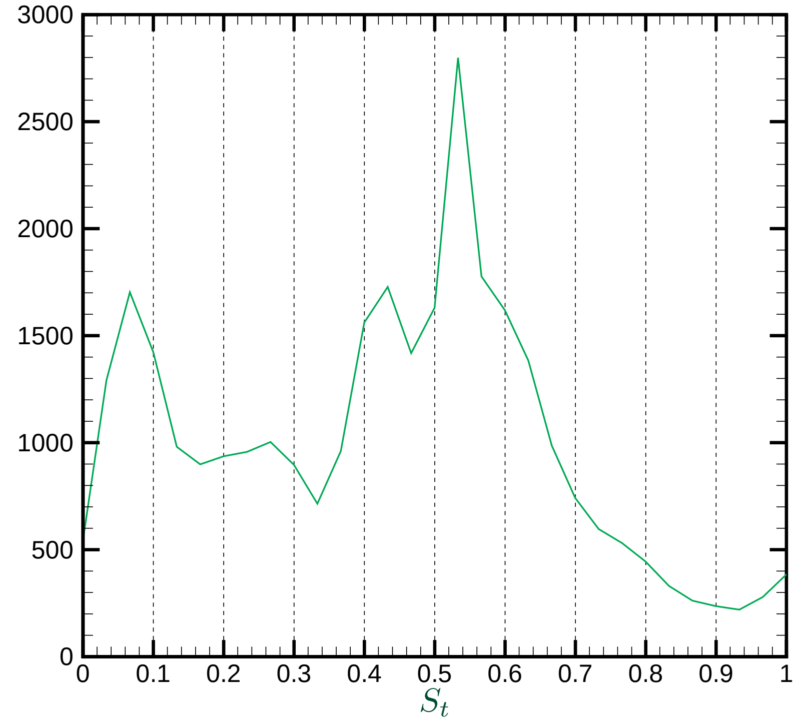
► Stationary cylinder $\gamma = 0$: $\hookrightarrow$ 2 *modes out of 100 are suffi cient to restitute* 97% *of the kinetic energy.*

► Controlled cylinder $\gamma = \gamma_e$: $\hookrightarrow$ 40 *modes out of 100 are then necessary to restitute* 97% *of the kinetic energy*

$\Rightarrow$ *Robustness evolution with dynamical evolutions.*

V - Closed loop results *Optimal control*





▶ Reduction of the wake instationarity. $\gamma_{opt} \simeq A\sin(2\pi S_t t)$ with $A = 2.2$ and $S_t = 0.53$

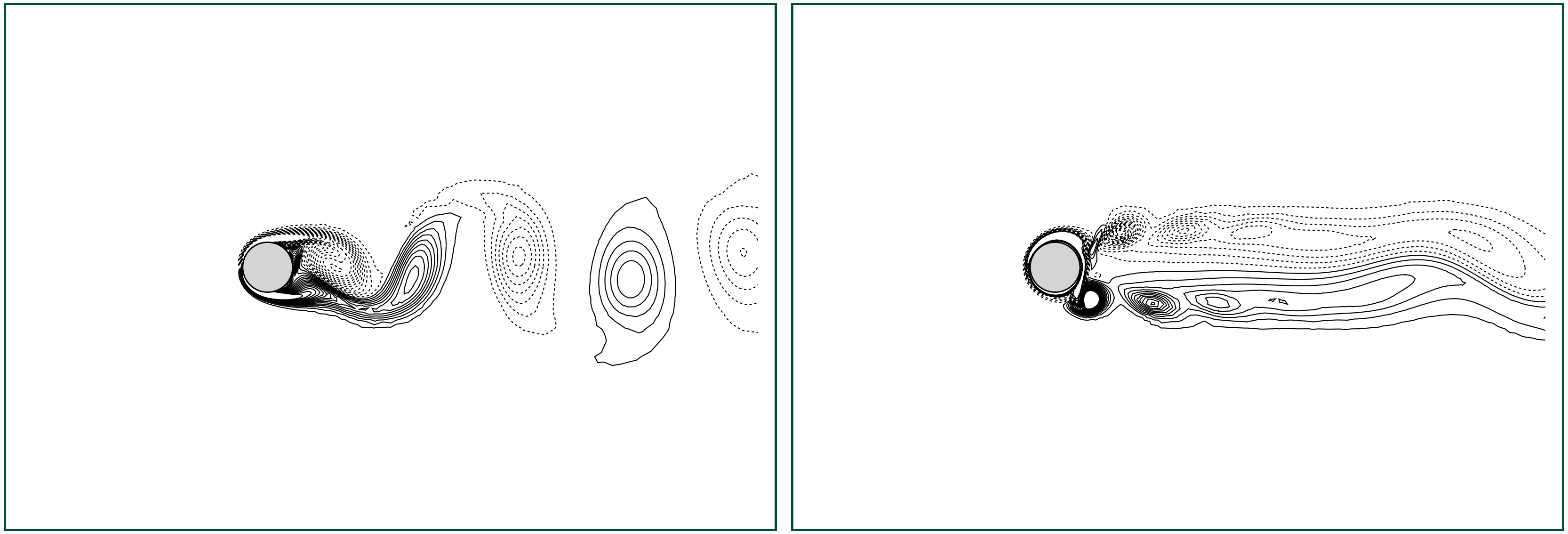
$$\mathcal{J}(\gamma_e) = 9.81 \quad \Longrightarrow \quad \mathcal{J}(\gamma_{opt}) = 5.63.$$

▶ The control is optimal for the reduced order model based on POD.

▶ Is it also optimal for the Navier Stokes model ?

V - Closed loop results *Comparison of wakes' organization*

- No mathematical proof concerning the Navier Stokes optimality.

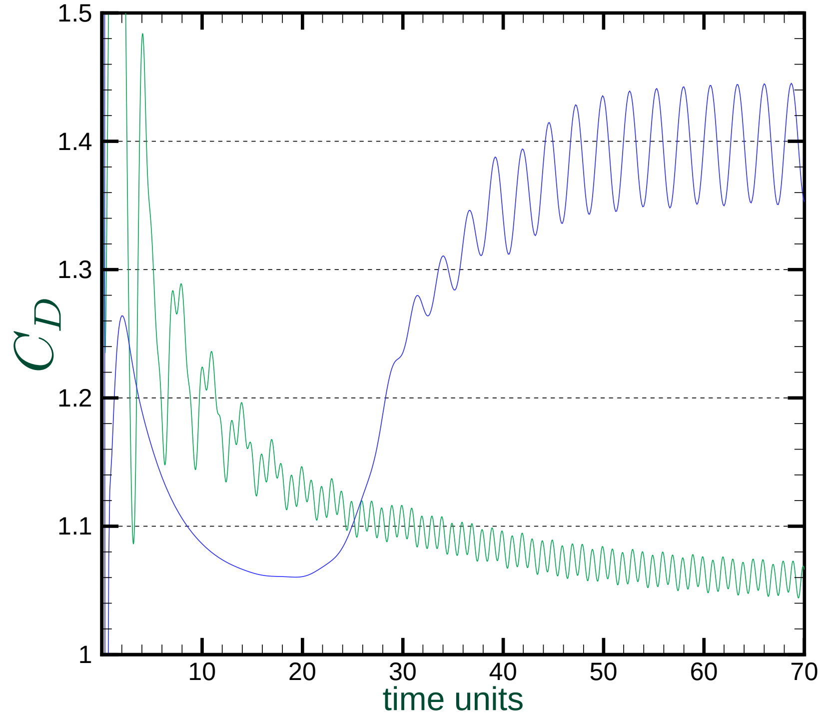


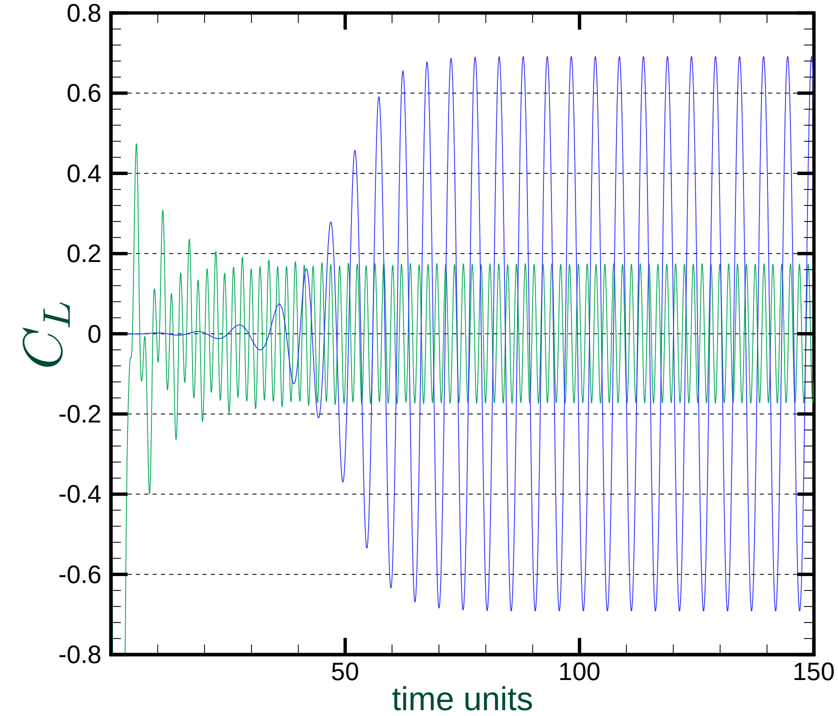
no control $\gamma = 0$ optimal control $\gamma = \gamma_{opt}$

Isocontours of vorticity ω_z.

- no control : $\gamma = 0 \Rightarrow$ Asymmetrical flow.
  $\hookrightarrow$ Large and energetic eddies.
- optimal control : $\gamma = \gamma_{opt} \Rightarrow$ Symmetrization of the (near) wake.
  $\hookrightarrow$ Smaller and lower energetic eddies.

V - Closed loop results *Aerodynamics coefficients*





- Very consequent drag reduction :

$$C_{D0} = 1.40 \text{ for } \gamma = 0 \text{ et } C_D = 1.04 \text{ for } \gamma = \gamma_{opt}$$
$$C_D/C_{D0} = 0.74 \Rightarrow \text{more than } 25\%.$$

- Decrease of the lift amplitude :

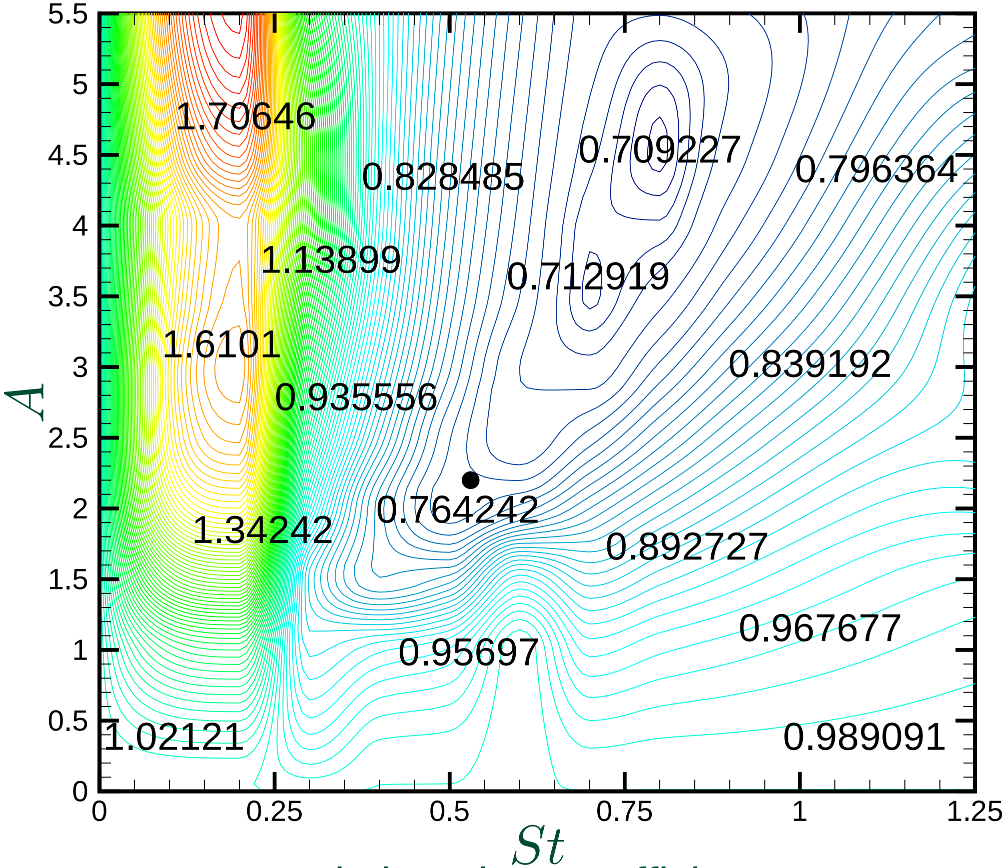
$$C_L = 0.68 \text{ for } \gamma = 0 \text{ et } C_L = 0.13 \text{ for } \gamma = \gamma_{opt}.$$

V - Closed loop results *Numerical costs*

- Optimal control of NSE by He *et al.* (2000) :
  $\hookrightarrow$ harmonic control law with $A = 3$ and $S_t = 0.75$.
  $\Rightarrow 30\%$ drag reduction.
- Optimal control POD ROM (this study) :
  $\hookrightarrow$ harmonic control law with $A = 2.2$ and $S_t = 0.53$.
  $\Rightarrow 25\%$ drag reduction.

- Less energetic costs (greater energetic gain ?)
- Calculus time costs : 100 less using POD ROM than NSE ! (for co-states equations and optimality conditions too)
- Memory storage : 600 less variables using POD ROM than NSE !

$\hookrightarrow$ "Optimal" control of 3D flows becomes possible !



Iso-relative- drag coefficient
$C_D(A,\ St)/C_{D0}$ *in* (A, St) *plan.*

Observations

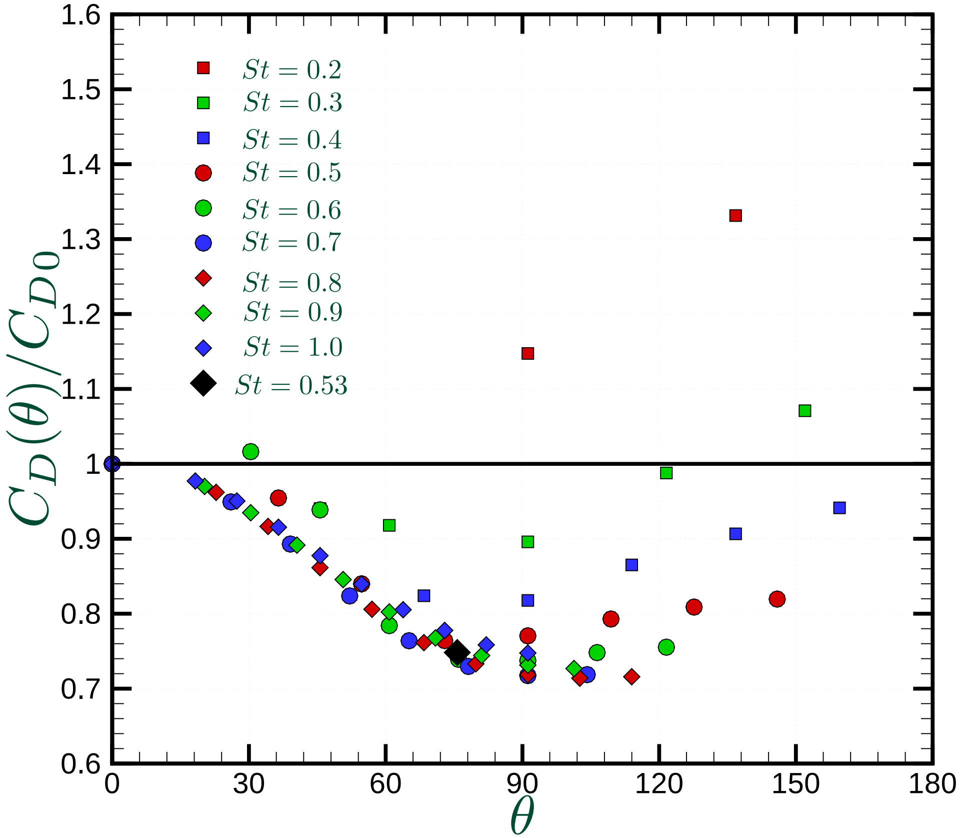
► Minimum is located in a smooth valley

$\hookrightarrow$ *Global minimum :*
around $A = 4.4$ *and* $St = 0.76$

► Maximum is located in a sharp pic

$\hookrightarrow$ *Global maximum :*
near $St = 0.2$*, the natural frequency : lock-on flow*

Finding the global minimum with an optimization algorithm may leads difficulties because of the smooth valley



Relative drag coefficient for different Strouhal numbers vs. maximum rotation angle.

- maximum rotation angle :

$$\theta = A/(\pi St)$$

Observations

- No drag reduction possible near natural frequency
- Maximum drag reduction around $\theta = \theta_{opt} = 95°$

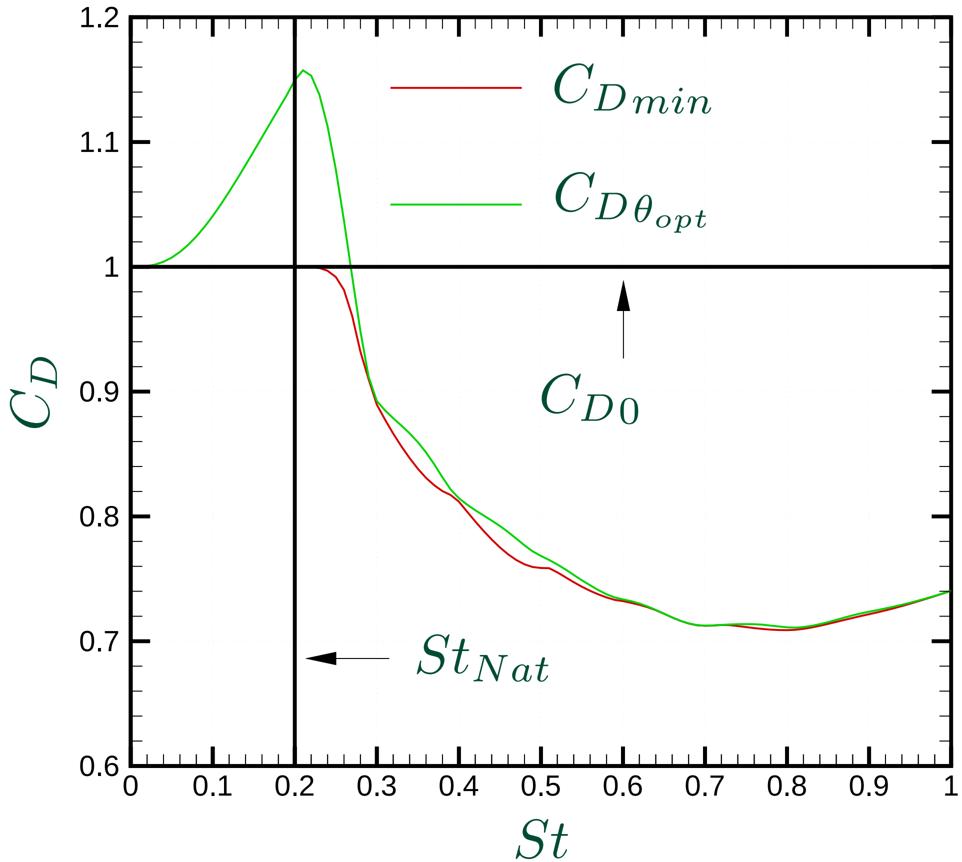
$\hookrightarrow$ *For all frequencies g.t. natural frequency*

$\hookrightarrow$ *Minimum drag :*

$$C_{D_{min}} = 0.71 C_{D0} = 0.98$$

Existence of an "optimal" maximal rotation angle.

VI - General observations *Maximal rotation angle*



Comparison between $C_{D\,min}$ and $C_{D\,\theta opt}$.

Notations

$$C_{D\,min}(St) = \min_{A\in\mathbb{R}} C_D(\theta,\ St)$$

$$C_{D\,\theta opt}(St) = C_D(95^\circ,\ St)$$

Observations

- Good agreements between $C_{D\,min}$ and $C_{D\,\theta opt}$ for $St > St_{Nat}$
- θopt is not optimal for $St < St_{Nat}$

In order to minimise the drag one couldn't choose A and St independently. It seems that $A/St = 5.2$ ($\theta_{opt} = 95^\circ$).

VI - General observations *Discussion*

- Control law obtain by POD ROM is not optimal for drag minimisation

  $\hookrightarrow$ Parameters obtain : $A = 2.2$ and $St = 0.53$ ($\theta_{max} = 76°$)

  $$\Rightarrow C_D = 1.04$$

  $\hookrightarrow$ Optimal parameters : $A = 4.4$ and $St = 0.76$ ($\theta_{max} = 105° \neq \theta_{opt}$)

  $$\Rightarrow C_D = 0.98$$

- Results in (A, St) quite different but not so far in C_D terms

  $\hookrightarrow$ The smooth valley is reached

- Couple an efficient new optimisation algorithm for smooth fonctions

  $\hookrightarrow$ Take results obtain by POD ROM as initial conditions

VII - Nelder Mead Simplex method *Generalities*

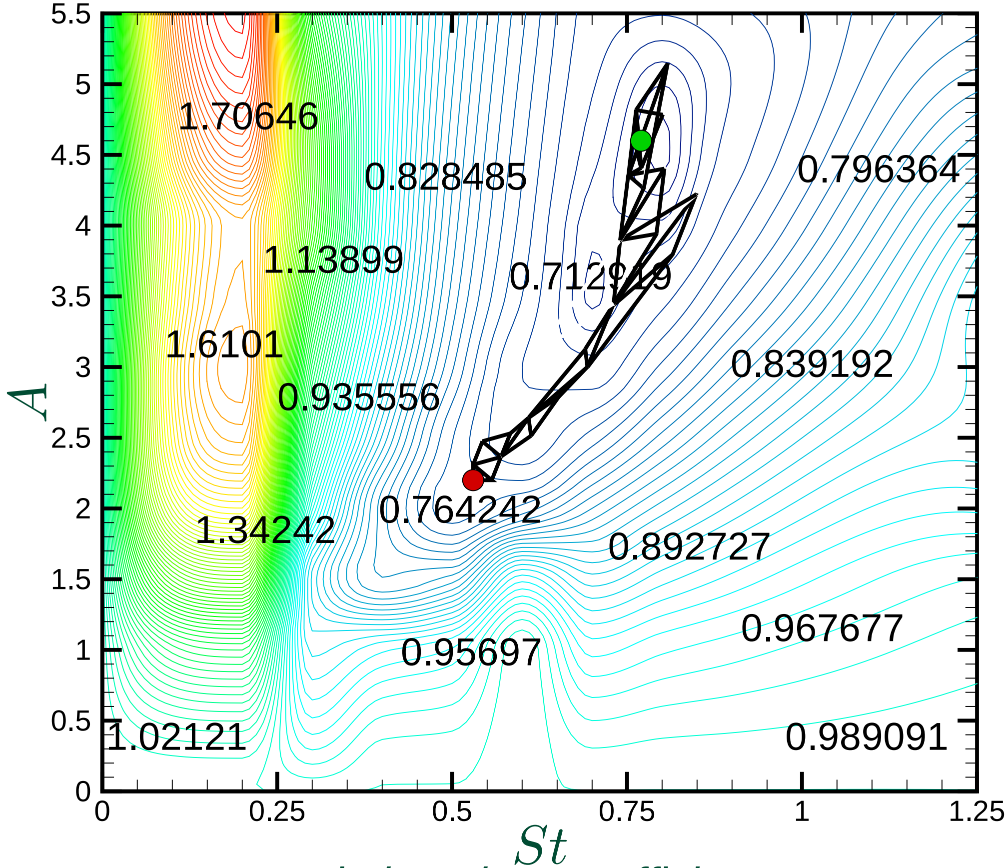
Avantages

- Numerical simplicities
- Adaptive topology
- Gradients calculations not necessary
- Good results with smooth fonctions

Disadvantages

- No proof of optimality for simplex dimensions greater than two
- Need to fix free parameters
- Maybe more iterations than gradient based optimisation algorithms...

VII - Nelder Mead Simplex method *Results*



Iso-relative- drag coefficient
$C_D(A,\ St)/C_{D0}$ *in* (A, St) *plan.*

Observations

▶ Topology adaptation function of the curve of the valley

▶ Minimum found by Nelder-Mead simplex method :
$A = 4.5$ and $St = 0.76 \Rightarrow \theta = 107°$
$\hookrightarrow$ Seems to be the global minimum

▶ 30 NSE resolutions $\Rightarrow$ 5% additive drag reduction compare to POD ROM

Relative drag reduction by POD ROM : 25% (1 NSE resolution)
Utility of coupling a new algorithm ?

Conclusions

- Significative drag reduction minimizing the wake instationnarity of the reduced order model : more than 25% of the relative drag
- But this is not a minimum (global a local) of the drag function
- Low calculus cost $\Rightarrow$ numerical cost negligible.
  $\hookrightarrow$ Calculus time costs : 100 less using POD ROM than NSE ! (for co-states equations and optimality conditions too)
  $\hookrightarrow$ Memory storage : 600 less variables using POD ROM than NSE !

  "OPTIMAL" CONTROL OF 3D FLOWS POSSIBLE BY POD ROM

- Existence of a optimal maximal rotation angle for effective drag reduction, $\theta = 105°$
- Couple POP ROM to Nelder Mead simplex method leads *a priori* the global minimum of the drag functional
- But the gain on the drag function is quite small compare to result obtain by POD ROM

Perspectives

- Improve the representativity of the low order model

  $\hookrightarrow$ "Optimize" the temporal excitation γ_e
  $\hookrightarrow$ Mix snapshots corresponding to several differents dynamics (temporal excitations)

- Look for harmonic control $\gamma(t) = A\sin(2\pi\, S_t\, t)$ with POD basis reactualization (close loop on NSE and not only on POD ROM)

- Couple this optimal system with trust region methods (TRPOD)
  $\Longrightarrow$ proof of convergence under weak conditions

- Couple pressure with the POD dynamical system

  $\hookrightarrow$ pressure contribution to drag coefficient : 80%

- Optimal control of the Navier Stokes equations