Quiz 4

MATH-UA.343, FALL 2017

SECTION 005, M. BILU

NAME / NETID:

Question 1.(4 points.) List the elements of $\mathfrak{A}_4$, the alternating group on four elements.

Question 2.(6 points.) Let σ_1 and σ_2 be the following permutations.

$$\sigma_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 1 & 15 & 2 & 14 & 3 & 13 & 4 & 12 & 5 & 11 & 6 & 10 & 7 & 9 & 8 \end{pmatrix},$$

$$\sigma_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 2 & 4 & 6 & 8 & 10 & 12 & 14 & 15 & 13 & 11 & 9 & 7 & 5 & 3 & 1 \end{pmatrix}.$$

1. Preliminary questions:
   (a) Decompose each permutation as a product of disjoint cycles.
   (b) Give the order of each permutation.
   (c) Give the sign of each permutation.

2. Compute the sign of

$$\sigma_1 \circ \sigma_2 \circ \sigma_1^{-4} \circ \sigma_2^3 \circ \sigma_1 \circ \sigma_2 \circ \sigma_1 \circ \sigma_2 \circ \sigma_1^{-1} \circ \sigma_2^{-6}.$$

Question 3.(6 points.) Prove that $\mathfrak{S}_n$ is generated by transpositions of the form $(1, i)$ with $i \in \{2, \ldots, n\}$ (remember that $\mathfrak{S}_n$ is generated by transpositions).

Question 4.(4 points.) Let $\sigma = (1, 3, 5, 6)$, $\tau = (2, 4, 7, 3)$ be two cycles of length 4 in $\mathfrak{S}_7$. Show that there exists a permutation α such that $\alpha \sigma \alpha^{-1} = \tau$.