Quiz 1

MATH-UA.343, FALL 2017

SECTION 005, M. BILU NAME:

Question 1.(4 points.) Give a proof of the Gauss lemma, stated below:
Let a, b, c be integers. If $a \mid bc$ and $gcd(a, b) = 1$ then $a \mid c$.

Question 2.(4 points.) Give the definition of an equivalence relation on a set A.

Problem 1.(6 points.) These questions are independent.

1. Find the inverse of 49 modulo 53.

2. Give the list of all the units of $\mathbb{Z}/15\mathbb{Z}$.

Problem 2.(6 points.) For any integer n, show that $\gcd(2n + 4, 3n + 3)$ can only be 1,2,3 or 6.

Bonus. For non-zero integers x, y, $\text{lcm}(x, y)$ is defined as the smallest positive common multiple of x and y.
Find all integers x, y such that: $gcd(x, y) + \text{lcm}(x, y) = x + y$.
(You can use the back of this sheet for this question)