Quiz 2

MATH-UA.343, FALL 2017

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Question 1.(4 points.) Give the definition of a group.

Question 2.(4 points.) Let $E = [0, 1]$ and let us define the following law on E:

$$\forall x, y \in E,\ x \star y = x + y - xy$$

Show that $\star$ is a law of composition. Is it associative? Commutative? Has it got an identity element?

Problem 1.(6 points.) Is B a subgroup of group A in these examples? Justify.

1. $A = (GL_2(\mathbf{R}), \cdot)$ and $B = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix},\ a, b \in \mathbf{R},\ a \neq 0 \right\}$.

2. $A = (GL_2(\mathbf{R}), \cdot)$ and $B = \left\{ \begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix},\ a \in \mathbf{R} \right\}$.

3. $A = (\mathbf{Q}^*, \times)$ and $B = \{2^n,\ n \in \mathbf{Z}\}$.

Problem 2.(6 points.) Below is a partially completed Cayley table of a **group**. Fill in the missing parts of the table.

$*$	a	b	c	d
a	b		d	
b	a			
c			b	
d				b