Quiz 5

Section 005, M. Bilu

Name / NetID:

Question 1.(4 points.) Let G be a group and let H be one of its subgroups.

1. State Lagrange's theorem.

2. What is the definition of the index of H in G?

Question 2.(6 points.)

1. $\{0, 4\}$ is a subgroup of $\mathbf{Z}/8\mathbf{Z}$. Write down its left cosets in $\mathbf{Z}/8\mathbf{Z}$.

2. $\{\mathrm{id}, (12)\}$ is a subgroup of $(\mathfrak{S}_3, \circ)$. How many right cosets does it have in $\mathfrak{S}_3$? Write them down.

Question 3.(6 points.) Give the list of the subgroups of $(\mathbf{Z}/15\mathbf{Z}, +)$.

Question 4.(4 points.) Let p be a prime number. Let G be a group of order p. Show that G is a cyclic group.

Bonus. Let $n \geq 2$ be an integer. Let's assume that for any $k \in \{1, \ldots, n-1\}$, $k^{n-1} \equiv 1 \pmod{n}$. Show that n is a prime number.