Functional inequalities for subellitpic heat kernels

In this thesis, I have studied the heat kernel and the associated functional inequalities on three model spaces in subelliptic geometry. These three spaces are 3-dimensional Lie groups: the Heisenberg group $\mathbb{H}$, $\mathbf{SU}(2)$ and $\mathbf{SL}(2,\mathbb{R})$. For each of this group, one can find a basis of the Lie algebra (X, Y, Z) which satisfies the relations:

$$[X, Y] = 2Z,\ [X, Z] = -2\rho Y \text{ and } [Y, Z] = 2\rho X$$

with $\rho = 1$ for $\mathbf{SU}(2)$, $\rho = 0$ for $\mathbb{H}$ and $\rho = -1$ for $\mathbf{SL}(2,\mathbb{R})$. As we will see it, this parameter ρ has an interpretation in mean of curvature. We then endow these three groups with the sublaplacian:

$$L = X^2 + Y^2$$

whehe we identify the matrices X, Y, Z with the left-invariant vector fields they engender. This is a left-invariant second order differential opertor. It is self-adjoint with respect to the Haar mesure of the group. It is not elliptic but hypoelliptic from Hörmander's results.

My results first concern the setting of explicit formulas for the above heat kernels. Then I focus on the generalisation in subelliptic geometry of the notion of Ricci curvature bounded below by a constant. In this way, I give a generalized Bakry-Emery curvature-dimension criterion which under some antisymetrical conditions satisfied by our model spaces, implies some Li-Yau type estimates. I also study the setting and the consequences of subcommutation inequalities between the gradient and the semigroup. In particluar, I give two new proofs of the H.Q.Li inequality on $\mathbb{H}$.

Keywords: heat semigroup, heat kernel, subelliptic geometry, Ricci curvature bounded below, functional inequalities, Poincaré inequality, Heisenberg group, Li-Yau estimates, Driver-Melcher inequality, H.Q.Li inequality, CR manifolds.