

# How a Genetic Algorithm Learns to Play Traveler's Dilemma by Choosing Dominated Strategies to Achieve Greater Payoffs

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- 2 Nash Equilibrium
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- 4 Experimental studies
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# Traveler's Dilemma

Scientific American Magazine June 2007

<http://www.scientificamerican.com/>

- The game was formulated in 1994 by Kaushik Basu.
- American Economic Review, Vol. 84, No. 2 May 1994.
- Scientific American Magazine, June 2007.



# Traveler's Dilemma

- An airline loses two suitcases belonging to two different travelers, Lucy and Pete. Both suitcases contain identical antiques.
- A manager tasked to settle the claims explains that the airline is liable for a maximum of \$100 per suitcase.
- The manager separates both travelers and asks them to write down the amount of their value, at no less than \$2 and no larger than \$100.
- If both the travelers write down the same number, the company reimburse both travelers that amount, otherwise the smaller number will be taken as the true dollar value...
- ... and both travelers will receive that amount along with a bonus /malus: + \$2 will be paid to the "honest" traveler and -\$2 to the person who wrote the highest price.

# Traveler's Dilemma

Question:

What number should the travelers write? What number would you write?

# Nash Equilibrium

## Nash Equilibrium

In game theory, the Nash equilibrium describes a kind of optimal strategy, informally defined as that set of strategies (one for each player) such that no player can do better by choosing a different strategy while keeping the others strategies fixed.

In a two player game, it would be a pair of strategies  $p, q$  such that:

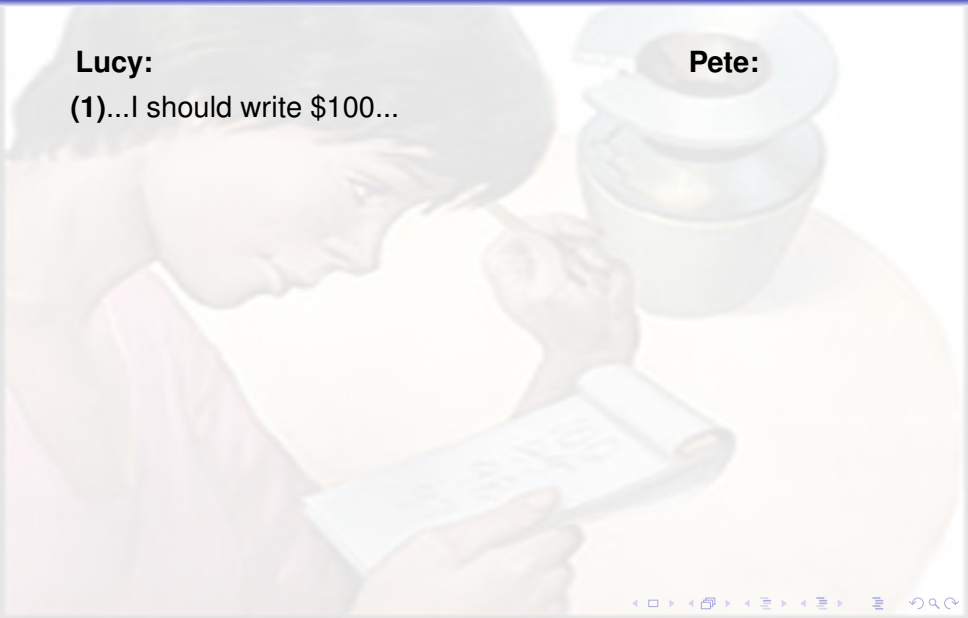
$$\begin{cases} \pi(p', q) \leq \pi(p, q) & \forall p' \neq p \\ \pi(p, q') \leq \pi(p, q) & \forall q' \neq q \end{cases}$$

# Nash Equilibrium for Traveler's Dilemma

**Lucy:**

**(1)**...I should write \$100...

**Pete:**



# Nash Equilibrium for Traveler's Dilemma

**Lucy:**

**Pete:**

(2)...I should write \$100..



# Nash Equilibrium for Traveler's Dilemma

**Lucy:**

**(3)**...if he thinks the same,  
then if I write \$99 I'll get a  
little bit more money: \$101

**Pete:**

# Nash Equilibrium for Traveler's Dilemma

**Lucy:**

**Pete:**

(4)... if she thinks I'll say \$100 she will probably say \$99, In that case I could do better by writing \$98, to have \$100..

# Nash Equilibrium for Traveler's Dilemma

**Lucy:**

**(5)**...if he thinks the \$98, I could deviate to \$97 and earn \$99..

**Pete:**

# Nash Equilibrium for Traveler's Dilemma

**Lucy:**

**Pete:**

**\$2!**

**\$2!**

**Nash Equilibrium!**

# Nash Equilibrium in Traveler's Dilemma

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# Nash Equilibrium in Traveler's Dilemma

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- Highly implausible that they would really go all the way down to 2. When the game is played experimentally most participants select much higher values, usually close to \$100.

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- 1965 Reinhard Selten proposed subgame perfect equilibrium to eliminate equilibria which depend on non-credible threats.

## What happens... really?

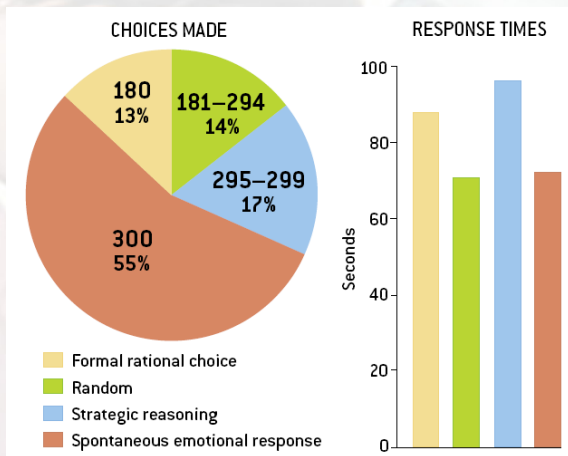
A celebrated lab experiment by Capra, Goeree, Holt and Gomez. (University of Virginia) with economics students and using real money:

- Choices between 80 and 200 cents.
  - Different penalties and rewards
  - The experiment confirmed the intuitive expectation...
  - Player would not play the Nash equilibrium strategy of 80c.
  - With a reward of 5 cents, the players' average choice was 180, falling to 120 when the reward rose to 80 cents.
- Web-based experiment confirmed.

# Why?

- The thought processes that produce this pattern of choices remain mysterious
- Altruism, socialization and faulty reasoning guide most individuals' choices

Rubinstein: 4 sets of possible choices for 4 different cognitive process:



## Link to Mr. Hingston presentation

- Iterated Prisoner's Dilemma for Species
- How can cooperation evolve?
- Evolution rewards the selfish.

## Repeated Traveler's Dilemma and G.A.

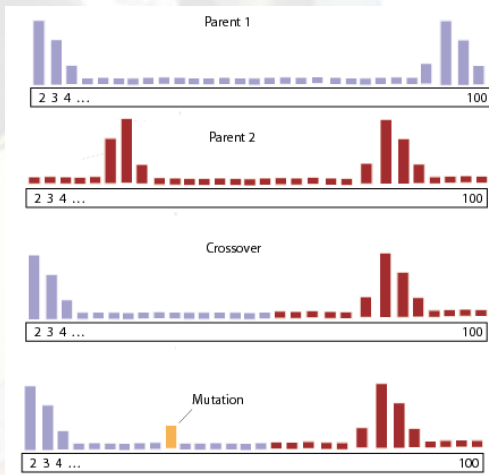
- A population of 100 players is set up.
- Each player chooses an answer for each match using a of probability distribution and plays against a random opponent.
- The best players (greater total payoff in the current generation) have a greater probability to be selected for the next generation.



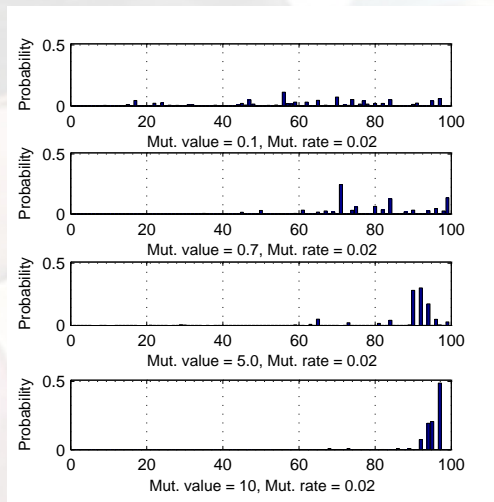
**Figure:** Example of a genome used in the G.A.

# Mutation and crossover

- Two individuals are chosen to mate.
- Their genomes are combined using a crossover operator.
- A gene is chosen randomly and a mutation value is added to it.



# Results



**Figure:** Average strategy after 5000 generations using different mutation values.

# Results

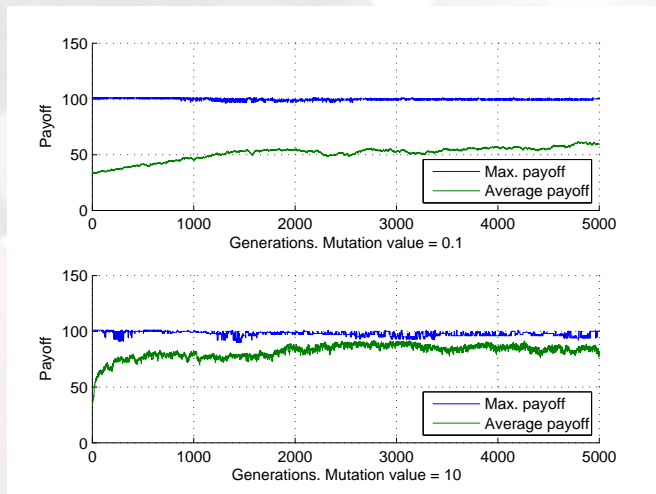
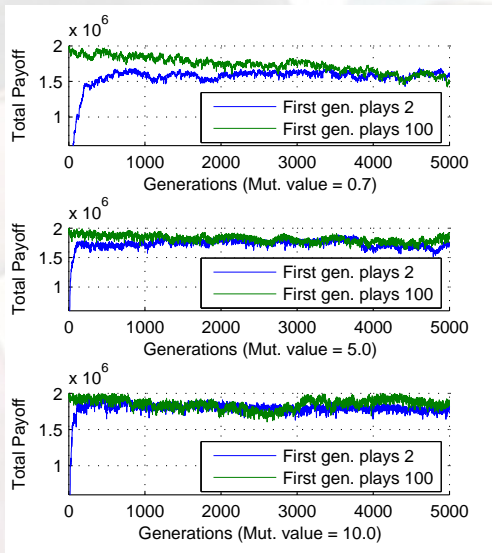


Figure: Average and maximal payoff for each generation.



# Convergence with different initial values

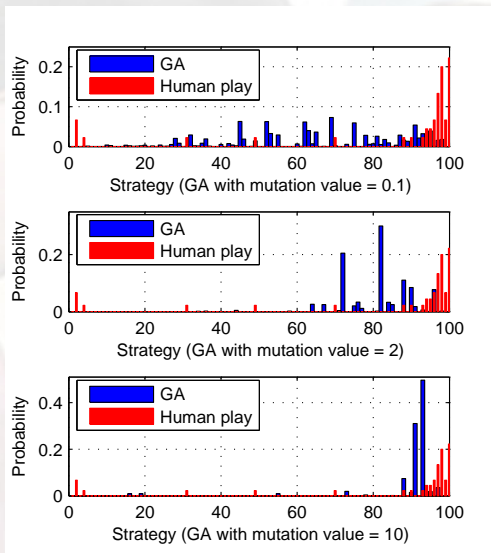


# GA vs Human strategy

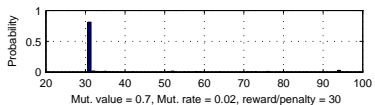
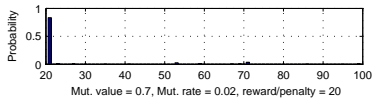
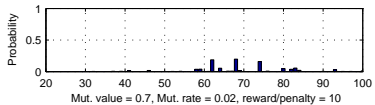
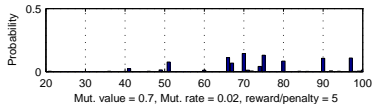
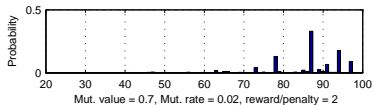
Strategy Entry		Strategy Entry		Strategy Entry	
2	3	88	1	96	3
4	1	90	1	97	6
31	1	93	1	98	9
49	1	94	2	99	3
70	1	95	2	100	10

**Table:** Distribution of human strategy described in Becker, Carter and Naeve. (Hohenheim)

# GA vs Human strategy




# Convergence varying rewards and penalties



# Conclusions

- In this paper, we proposed a genetic algorithm to search over the probability space for the distributions that maximize the average payoff in repeated game sessions.
- Even if changing the rewards and penalties should theoretically have no impact, in practice, it has. The algorithm showed the same behaviour without having any a priori knowledge about the game. This probably means that the reasons of this effect are not exclusively psychological.
- The results show forms of convergence to equilibrium distributions.
- Possibility to apply probabilistic methods and convergence analysis.

# Conclusions

A young girl with dark hair, wearing a pink shirt, is sitting at a table. She is looking down at a small, open notebook she is holding in her hands. She has a pen in her right hand, resting it on the notebook. On the table in front of her is a white bowl filled with a golden-brown, bubbly substance, possibly a snack or dessert. The background is a plain, light-colored wall.

Thank you!