GdR MIA Thematic day on Non-Convex Sparse Optimization

Friday October 9th 2020 at ENSEEIHT, Toulouse

Summary

Sparse models are widely used in machine learning, statistics, and signal/image processing applications. They usually lead to NP-hard non-convex optimization problems that involve the ℓ_0 pseudo-norm. The purpose of this thematic day is to bring together speakers from different teams in order to explore the recent progress made in solving these challenging non-convex optimization problems. We expect to cover a variety of methods that include (but not only) greedy algorithms, continuous relaxations, screening rules, as well as global optimization through branch-and-bound strategies or semidefinite programming.

Call for contributions

If you wish to present your work, please send your proposal by (title, authors, affiliation, abstract) to **emmanuel.soubies@irit.fr** by **July 31th 2020**. Depending on the number of contributions, a poster session may be organised.

Registration

Registration is free but **mandatory**.
To register, please fill the form (Before **September 27, 2020**): `https://evento.renater.fr/survey/registration-to-gdr-mia-thematic-day-on-non-convex-sparse-optimization-l5d61xdy`

Format and Venue

This thematic day will take place in the engineering school INP-ENSEEIHT (2, rue Charles Camichel, 31000 Toulouse), located in the center of Toulouse.
Virtual participation will also be possible via Zoom (the link will be sent by email to participants).

Organizer

Emmanuel Soubies (CNRS, IRIT, Université de Toulouse). Mail: emmanuel.soubies@irit.fr

Program

Chair:	Emmanuel Soubies	
08h30 – 09h00	Manlio Gaudioso	*A k-norm-based Mixed Integer Programming formulation for sparse optimization.*
09h00 – 09h30	Sébastien Bourguignon	*Exact ℓ_0-norm optimization via branch-and-bound methods*
09h30 – 10h00	Arthur Marmin	*Global solution to non-convex optimization problems involving an approximate ℓ_0 penalization.*
10h00 – 10h30	*Coffee break*	
Chair:	Cássio Fraga Dantas	
10h30 – 11h00	Vasiliki Stergiopoulou	*Microscopic super-resolution in the lateral plane by sparse regularization in the correlation domain*
11h00 – 11h30	Fiorella Sgallari	*Convex non-convex variational models for image processing*
11h30 – 12h00	Joseph Salmon	*Safe screening and active sets for sparse regularization*
12h00 – 14h00	*Lunch*	
Chair:	Cédric Févotte	
14h00 – 14h30	Charles Soussen	*Exact recovery analysis of non-negative orthogonal greedy algorithms*
14h30 – 15h00	Liva Ralaivola	*Recovery and convergence rate of the Frank–Wolfe algorithm for the m-exact-sparse problem*
15h00 – 15h30	Yann Traonmilin	*A framework for non-convex recovery of low dimensional models in infinite dimension.*
15h30 – 16h00	*Coffee break*	
Chair:	Henrique Goulart	
16h00 – 16h30	Léon Zheng	*Identifiability in matrix sparse factorization*
16h30 – 17h00	Nicolas Nadisic	*Sparse separable nonnegative matrix factorization*

Abstracts

A k-norm-based Mixed Integer Programming formulation for sparse optimization.

Manlio Gaudioso *(DIMES-Università della Calabria)*

We introduce a mixed integer programming (MIP) formulation of the sparse optimization problem where the objective is to minimize the weighted sum of a convex (not necessarily differentiable) function and the ℓ_0 pseudo-norm of the vector of variables. Our formulation is based on some properties of the k-norm, which is the sum of k largest components (in modulus) of a vector. We focus, in particular, on the continuous relaxation of the model, which results in a Difference of Convex (DC) optimization problem. The approach is numerically tested within the framework of Feature Selection in Support Vector Machine (SVM), a well known sparse optimization problem in Machine Learning. Comparison with standard ℓ_1-SVM on some benchmark problems is also provided.

Joint work with Giovanni Giallombardo and Giovanna Miglionico (DIMES-Università della Calabria).

Exact ℓ_0-norm optimization via branch-and-bound methods.

Sébastien Bourguignon *(École Centrale de Nantes)*

We propose exact optimization algorithms for solving least-squares problems with low cardinality (small ℓ_0 "norm"). Such problems arise in many applications, including subset selection in statistics, inverse problems in signal processing or portfolio optimization. Although NP hard, these problems have been tackled from a global optimization perspective, considering a mixed integer programming (MIP) reformulation with binary variables. Resolution is then performed via branch-and-bound methods, which implicitly explore a full combinatorial search tree, and therefore guarantee the optimality of the solution. In this talk, we show that such algorithms can be designed without resorting to MIP formulations, and that the relaxation problems involved at each node of the search tree can be reformulated as ℓ_1-norm-based continuous optimization problems, for which a dedicated algorithm is built. A specific tree exploration strategy is proposed. The resulting procedure is shown to outperform the CPLEX MIP solver, being able to solve exactly (with optimality proof) problems involving 1000 unknowns and up to a few tens of nonzero components in less than 1000 seconds. When their computation is possible, solutions are shown to improve over state-of-the-art sparsity-enhancing algorithms with lower computational complexity.

Joint work with Ramzi Ben Mhenni (École Centrale de Nantes), Jordan Ninin (Lab-STICC,ENSTA-Bretagne), Marcel Mongeau (Univ. de Toulouse, ENAC) et Hervé Carfantan (Univ. de Toulouse, UPS, CNRS, CNES, IRAP).

Global solution to non-convex optimization problems involving an approximate ℓ_0 penalization

Arthur Marmin *(Univ. Paris-Saclay, CentraleSupélec, Centre de Vision Numérique, INRIA)*

For dealing with sparse models, a large number of continuous approximations of the ℓ_0 penalization have been proposed. However, the most accurate ones lead to non-convex optimization problems. By observing that many such approximations are piecewise rational functions, we show that the original optimization problem can be recast as a multivariate polynomial problem. The latter is then globally solved by using recent optimization methods which consist of building a hierarchy of convex problems. Our methodology is then applied on a reconstruction problem inspired by chromatography. Experimental results illustrate that our method provides a global optimum of the initial problem for standard ℓ_0 approximations. This is in contrast with existing local algorithms whose results depend on the initialization.

Joint work with Marc Castella (SAMOVAR, Télécom SudParis, Institut Polytechnique de Paris) and Jean-Christophe Pesquet (Univ. Paris-Saclay, CentraleSupélec, Centre de Vision Numérique, INRIA).

Microscopic super-resolution in the lateral plane by sparse regularization in the correlation domain.

Vasiliki Stergiopoulou *(Univ. Côte d'Azur, I3S, CNRS, Inria)*

Super-resolution light microscopy overcomes the diffraction barrier as it allows the observation of subcellular entities invisible otherwise. Various super-resolution methods have been developed during the last years. In our work, we consider a method ideal for live-cell super-resolution imaging, allowing sufficient temporal resolution, by means of common microscopes (standard widefield, TIRF, scanning confocal, etc.) and conventional fluorescent dyes. Our model exploits the assumption of sparse distribution of the fluorescent molecules as well as the temporal and spatial independence of the emitters. Mathematically, this is achieved by formulating a variational model where the sparsity of the spatio-temporal covariance matrix is achieved by using the ℓ_0 pseudo-norm as regularization term. To avoid dealing with such NP-hard non-convex problems, we consider the CEL0 relaxation of the ℓ_0 norm. In order to deal with real data of biological interest, parameters such as the background and the variance of noise have further to be estimated. By including a further, computationally cheap, optimization step, the proposed method is also capable of retrieving intensity information in the estimated support, which can be a very valuable information for 3D super-resolution imaging

Joint work with Luca Calatroni, Laure Blanc-Féraud (Univ. Côte d'Azur, I3S, CNRS, Inria), Sébastien Schaub (Univ. Côte d'Azur CNRS, IBV), and José Henrique de Morais Goulart (Univ. de Toulouse, IRIT/INP-ENSEEIHT).

Convex non-convex variational models for image processing.

Fiorella Sgallari *(University of Bologna)*

An important class of computational techniques to solve inverse problems in image processing relies on a variational approach: the optimal output is obtained by finding a minimizer of an energy function or "model" composed of two terms, the data-fidelity term and the regularization term. Much research has focused on models where both terms are convex, which leads to convex optimization problems. However, there is evidence that non-convex regularization can improve significantly the output quality for images characterized by some sparsity property. This fostered recent research towards the investigation of optimization problems with non-convex terms. Non-convex models are notoriously difficult to handle as classical optimization algorithms can get trapped at unwanted local minimizers. To avoid the intrinsic difficulties related to non-convex optimization, the convex non-convex (CNC) strategy has been proposed, which allows the use of non-convex regularization while maintaining convexity of the total cost function.

In this talk we focus on a general class of parameterized non-convex sparsity-inducing separable and non-separable regularizers and their associated CNC variational models. Convexity conditions for the total cost functions and related theoretical properties are discussed, together with suitable algorithms for their minimization based on a general Forward-Backward (FB) splitting strategy. Experiments on the two classes of considered separable and non-separable CNC variational models show their superior performance than the purely convex counterparts when applied to the discrete inverse problem of restoring sparsity-characterized images corrupted by blur and noise.

Joint work with Alessandro Lanza (University of Bologna), Serena Morigi Serena Morigi and Ivan Selesnick (New York University).

Safe screening and active sets for sparse regularization.

Joseph Salmon *(IMAG, Univ. de Montpellier)*

Owing to their statistical properties, non-convex sparse regularizers have attracted much interest for estimating a sparse linear model from high dimensional data. Given that the solution is sparse, for accelerating convergence, a working set strategy addresses the optimization problem through an iterative algorithm by incrementing the number of variables to optimize until the identification of the solution support. We will review such methods for convex regularizers, and their connexions with safe screening rules. Then, we will investigate the interest of such concepts for non-convex sparse regularizers.

Joint work with Alain Rakotomamonji (LITIS, Univ. de Rouen, Criteo AI Lab), Gilles Gasso (LITIS, INSA de Rouen), Rémi Flamary (Univ Cote d'Azur, OCA Lagrange) (non-convex case) and with Mathurin Massias (MaLGa, DIBRIS, Università degli Studi di Genova) and Alexandre Gramfort (Univ. Paris-Saclay, Inria, CEA) (convex case).

Recovery and convergence rate of the Frank–Wolfe algorithm for the m-exact-sparse problem.

Liva Ralaivola *(Criteo)*

In this work, we study the properties of the Frank-Wolfe algorithm to solve the m-EXACT-SPARSE reconstruction problem. We prove that when the signal y is sparse enough with respect to the coherence of the dictionary, then the iterative process implemented by the Frank-Wolfe algorithm only recruits atoms from the support of the signal (the support being the smallest set of atoms of the dictionary that allows for a perfect reconstruction of y). We also prove that under this same condition, there exists an iteration beyond which the convergence rate of the algorithm is exponential.

Joint work with Farah Cherfaoui (LIS, Aix-Marseille Université, CNRS), Valentin Emiya (LIS, Aix-Marseille Université, CNRS), and Sandrine Anthoine (I2M, Aix-Marseille Université, CNRS).

Exact recovery analysis of non-negative orthogonal greedy algorithms.

Charles Soussen *(L2S, CentraleSupélec, Univ. Paris-Saclay)*

Many application fields give rise to inverse problems where the signal or image of interest is sparse, but also non-negative. We introduce a family of greedy algorithms for non-negative sparse reconstruction, generalizing the classical Orthogonal Matching Pursuit and Orthogonal Least Squares algorithms. Taking the non-negativity constraint into account yields important challenges both in terms of fast implementation and of K-step exact recovery analysis. We present novel contributions regarding both issues. Regarding the second topic, we show that for incoherent dictionaries, K-sparse non-negative representations can be recovered in K steps with the generalized versions of OMP and OLS. We further discuss the main challenges of deriving improved analyses for correlated dictionaries.

Joint work with Thanh T. Nguyen, El-Hadi Djermoune (Univ. de Lorraine, CNRS, CRAN), and Jérôme Idier (CNRS, LS2N) .

A framework for non-convex recovery of low dimensional models in infinite dimension.

Yann Traonmilin *(Institut de Mathématiques de Bordeaux)*

Non-convex methods for linear inverse imaging problems with low-dimensional models have emerged as an alternative to convex techniques. We propose a theoretical framework where both finite dimensional and infinite dimensional linear inverse problems can be studied. This framework recovers existing results about low-rank matrix factorization and off-the-grid sparse spike estimation, and it provides new results for Gaussian mixture estimation from linear measurements.

Joint work with Jean-François Aujol and Arthur Leclaire (Institut de Mathématiques de Bordeaux).

Identifiability in matrix sparse factorization.

Léon Zheng *(Inria DANTE, ENS de Lyon (LIP))*

Matrix sparse factorization is a multilinear inverse problem where given an observed matrix $\mathbf{Z}$ and some sparsity constraints, one tries to recover some sparse factors for which the matrix product is equal to $\mathbf{Z}$. In order to better understand how to design provably-good algorithms for matrix sparse factorization, this work provides some identifiability results in the specific case of matrix factorization with only two factors, i.e., some conditions for which the observation $\mathbf{Z}$ is sufficient to recover the pair of sparse factors $(\mathbf{X}, \mathbf{Y})$ for which $\mathbf{XY} = \mathbf{Z}$, up to unavoidable permutation and scaling ambiguities due to the nature of matrix product. In particular, this work analyzes two important prob- lem instances: the case where one of the two factors is fixed, and the case where the supports of the pair of factors is fixed. In the second case, one important characterization of entry values identifiability is iterative completability, i.e., the fact that the rank 1 matrices induced by the product between one column of the left factor and one row of the right factor can be completed one by one. Analyz- ing these two specific instances allows us to establish some important necessary conditions for identifiability in the general case, which can lead in a future work to general conditions of identifiability in matrix sparse factorization.

Joint work with Rémi Gribonval (Inria DANTE, ENS de Lyon (LIP)).

Sparse separable nonnegative matrix factorization.

Nicolas Nadisic *(University of Mons)*

We propose a new variant of nonnegative matrix factorization (NMF), combining separability and sparsity assumptions. Separability requires that the columns of the first NMF factor are equal to columns of the input matrix, while sparsity requires that the columns of the second NMF factor are k-sparse. We call this variant sparse separable NMF (SSNMF), which we prove to be NP-complete, as opposed to separable NMF which can be solved in polynomial time. The main motivation to consider this new model is to handle underdetermined blind source separation problems, such as multispectral image unmixing. We introduce an algorithm to solve SSNMF, based on the successive nonnegative projection algorithm (SNPA, an effective algorithm for separable NMF), and an exact sparse nonnegative least squares solver. We prove that, in noiseless settings and under mild assumptions, our algorithm recovers the true underlying sources. This is illustrated by experiments on synthetic data sets and the unmixing of a multispectral image.

Joint work with Arnaud Vandaele, Nicolas Gillis (University of Mons) and Jérémy E. Cohen (Univ Rennes, Inria, CNRS, IRISA).