Operations research:
Mathematics and algorithmics for solving decision-making problems

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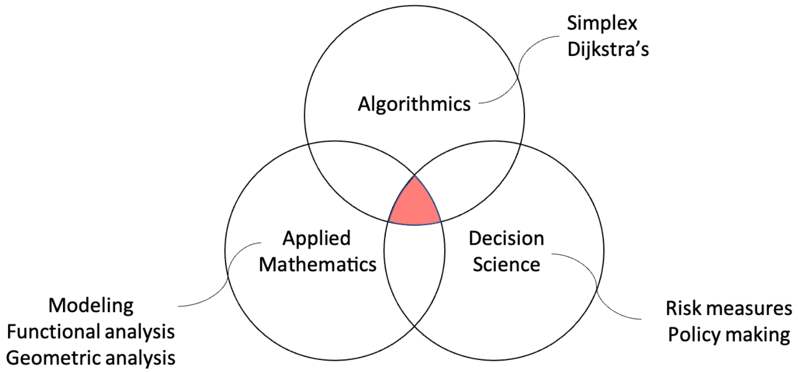
Operations research (OR)

- Operations research and optimization are at the intersection of multiple disciplines.



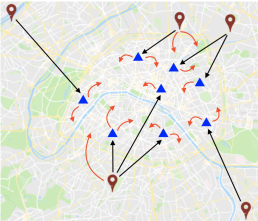
Operations research (OR)

- OR solves optimization problems that decision-makers (managers, politicians, engineers, etc.) encounter.
- Problems are typically expressed in terms of decisions, costs and constraints.
- Tools coming from mathematics, informatics, economics and industrial engineering are often used in their solution.
- The end result is a decision-making tool.

$$\min_{\mathbf{x} \in X} f(\mathbf{x})$$

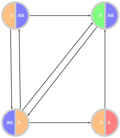
Some examples

- Route optimization
- Planning and scheduling
- Network design (telecommunications, distribution, electricity, etc.)
- Supply chain management



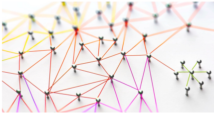
Some projects from our team[1]

Kidney exchange



Maintenance planning

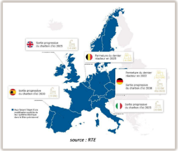
Retail network design



Phytosanitary treatments

Maritime transportation

Energy mix planning



[1] Team EDGE-Centre Inria de l'Universite de Bordeaux

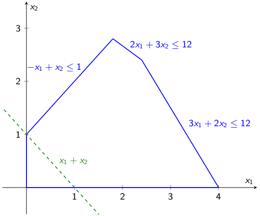
Classical tools of operations research[2]

- Mathematical programming
  - Linear programming (LP/PL)
  - Mixed-integer linear programming (MILP/PLNE)
  - ..
- Graph theory and algorithms
- Constraint programming (CP/PPC)
- Convex analysis
- Approximation algorithms
- Heuristics, metaheuristics
- Queueing theory, simulation, statistics
- ...

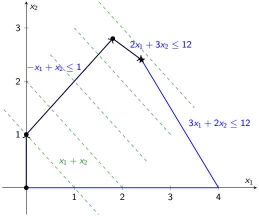
[2] Most topics are covered in the master MAS-ROAD

Mathematical programming: Linear programming

$$\min_{\mathbf{x}\in\mathbb{R}^n_+} \quad \mathbf{c}^\top \mathbf{x}$$
$$\text{s.t.} \quad \mathbf{A}\mathbf{x} \leq \mathbf{b}$$



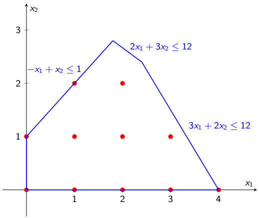
- Well-known algorithms: Simplex, ellipsoid, interior point, etc.
- Solvers: CPLEX, Gurobi, Clp, HiGHS, Excel-Solver etc.



$$z = \mathbf{c}_\mathbf{B}^\top \mathbf{A}_\mathbf{B}^{-1}\mathbf{b} + \left(\mathbf{c}_\mathbf{N} - \mathbf{c}_\mathbf{B}^\top \mathbf{A}_\mathbf{B}^{-1}\mathbf{A}_\mathbf{N}\right)^\top \mathbf{x}_N$$
$$\mathbf{x}_\mathbf{B} = \mathbf{A}_\mathbf{B}^{-1}\mathbf{b} - \mathbf{A}_\mathbf{B}^{-1}\mathbf{A}_\mathbf{N}\mathbf{x}_\mathbf{N}$$

Mathematical programming: Mixed-integer programming

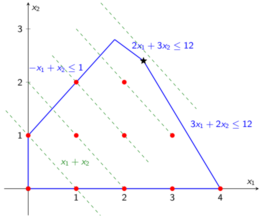
$$\min_{\mathbf{x} \in \mathbb{Z}_+^{n_1} \times \mathbb{R}_+^{n_2}} \quad \mathbf{c}^\top \mathbf{x}$$
$$\text{s.t.} \quad \mathbf{A}\mathbf{x} \leq \mathbf{b}$$

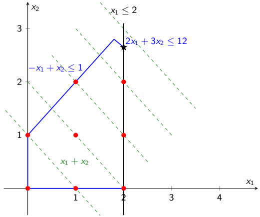


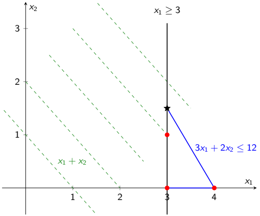
- Integer/binary decisions that cannot be rounded from fractions:
  - Do we open facility i or not?
  - How many trucks do we need to send to client j from facility i?
  - If facility i is not open then it cannot be used to satisfy demand.
- Well-known algorithms: Branch & Bound, Branch & Cut etc.
- Solvers: CPLEX, Gurobi, HiGHS, GLPK etc.

Mathematical programming: Mixed-integer programming

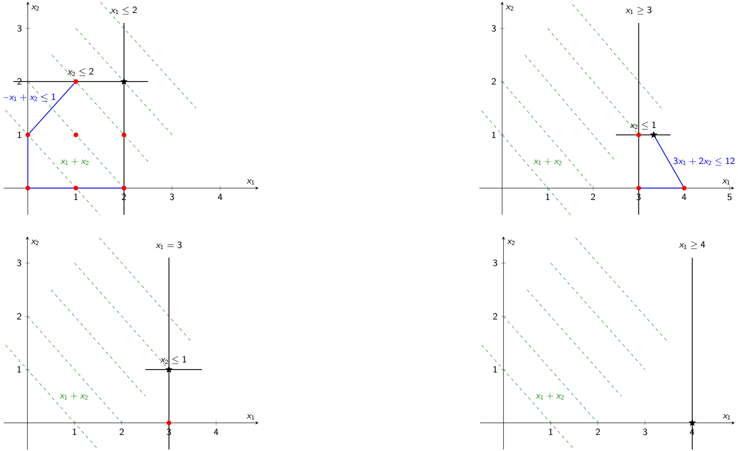
- Branch & Bound: Solve relaxations and successively partition the feasible region







Mathematical programming: Mixed-integer programming



Difficulty of mixed-integer programming problems[3]

- Mixed-integer programming problems are NP-Complete in the general case.
- In practice, considerable progress has been made since the beginning.

A classical problem: Traveling salesman

- Find the shortest cycle passing through N cities given the pairwise distances.
- In theory finding the best cycle requires testing $N!$ possibilities.
  - For 10 cities (N=10) : $<$ 1 milliseconds
  - For 30 cities (N=30) : 35000 billion years

[3] Source : http://www.math.uwaterloo.ca/tsp/

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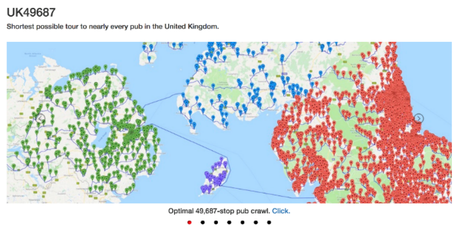

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Difficulty of mixed-integer programming problems[3]

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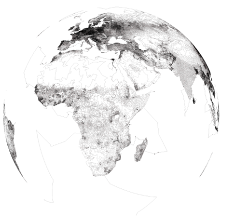
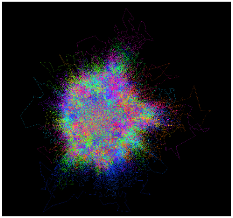
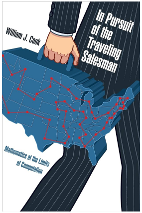
1.9 millions cities to 0.0473% optimality



1 331 906 450 stars to 0.37% optimality



- Powerful heuristics coupled with branch & cut
- Many results dedicated to the problem
- Months of computation in parallel processing



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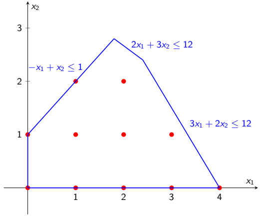
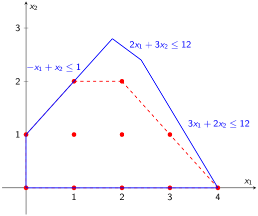
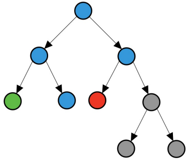
Advanced techniques in mixed-integer programming

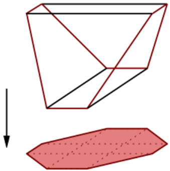




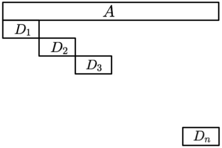
Implicit enumeration



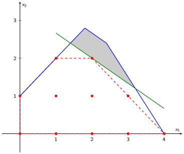
Extended formulations



Decomposition algorithms



Geometric analysis



Conclusions

- OR is an interdisciplinary field that studies decision-making problems from a mathematical and algorithmic perspective.
- It develops tools to help decision makers.
- Significant algorithmic progress has been made in recent years.
- Further research is needed in order to extend classical results to more realistic contexts.

Questions?

Thank you for your attention!
Any questions?

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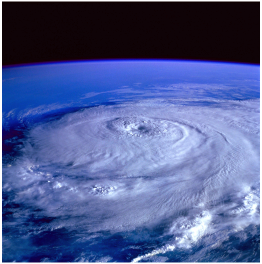
Appendix:
Optimisation under uncertainty

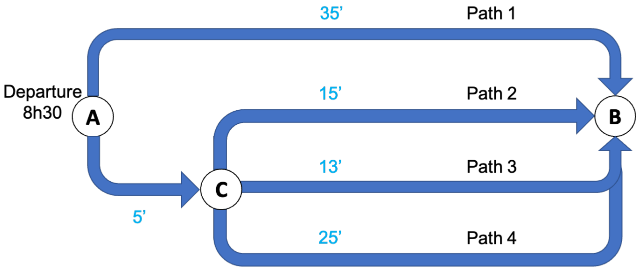
Presence of uncertainty

- New challenges:
  - Renewable energy production.
  - Resilient network design.
  - Healthcare/disaster management.
  - Circular economy.
  - Security and defense.



- Uncertainty:
  - Stochastic nature of systems.
  - Long duration of decision processes.
  - Difficulty of precise measurements.
  - Lack of historical information.
  - Presence of adversarial participants.

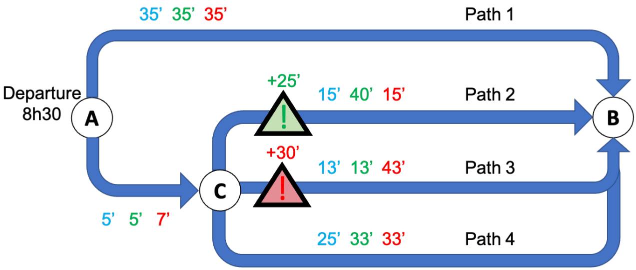
A practical example: Shortest path under uncertainty



Path 1	35'
Path 2	20'
Path 3	18'
Path 4	30'

- What is the shortest path from point A to point B?

A practical example: Shortest path under uncertainty

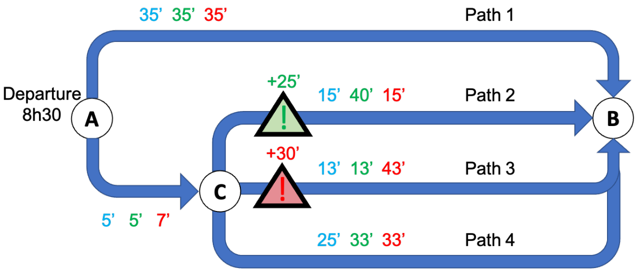


Scenario	Probability
1	0,5
2	0,25
3	0,25

- What is the shortest path from point A to point B?

Our first order of business is to characterize the uncertain data.

A practical example: Shortest path under uncertainty



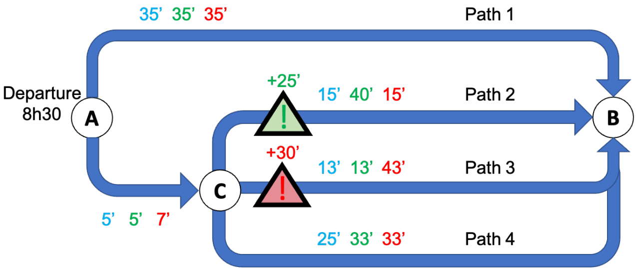
Scenario	Probability
1	0,5
2	0,25
3	0,25

Scenario	1	2	3
Path 1	35'	35'	35'
Path 2	20'	45'	22'
Path 3	18'	18'	50'
Path 4	30'	38'	40'

- What is the shortest path from point A to point B?

Second order of business is to characterize what constitutes a "good" solution.

A practical example: Shortest path under uncertainty



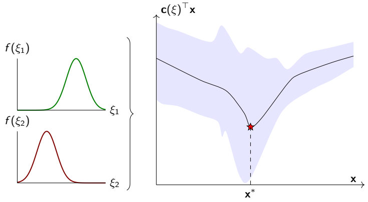
Scenario	Probability
1	0,5
2	0,25
3	0,25

Scenario	1	2	3	Esp	Max
Path 1	35'	35'	35'	35'	35'
Path 2	20'	45'	22'	26,75'	45'
Path 3	18'	18'	50'	26'	50'
Path 4	30'	38'	40'	34,5'	38'

In optimization under uncertainty the notion of a good solution depends on the risk preferences of the decision-maker.

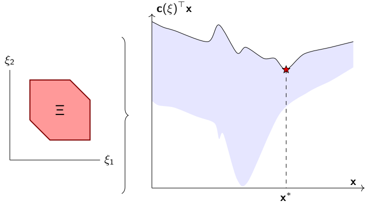
Optimization under uncertainty paradigms

Stochastic optimization



- Distribution $\mathbb{P}$ is known.
- Consequences are observed repeatedly.
- Risk level is low or moderate.
- Example: Distribution network design.

Robust optimization



- Distribution is not known (or no distribution).
- Consequences are observed once.
- Risk level is high.
- Example: Disaster management.

Optimization under uncertainty paradigms

Stochastic optimization

$$\begin{aligned} \min_{\mathbf{x}\in X} \quad & \mathbb{E}^{\mathbb{P}}_{\xi\in\Xi}\left[\mathbf{c}(\xi)^\top \mathbf{x}\right] \\ \text{s.t.} \quad & \mathbf{A}\mathbf{x} \leq \mathbf{b} \end{aligned}$$

- Distribution $\mathbb{P}$ is known.
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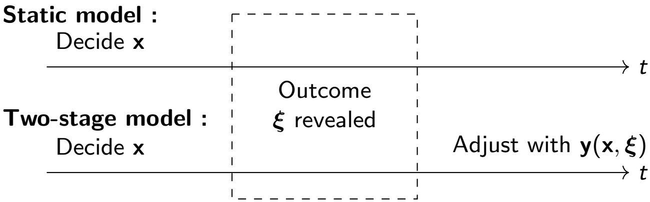
Robust optimization

$$\begin{aligned} \min_{\mathbf{x}\in X} \quad & \max_{\xi\in\Xi}\left[\mathbf{c}(\xi)^\top \mathbf{x}\right] \\ \text{s.t.} \quad & \mathbf{A}\mathbf{x} \leq \mathbf{b} \end{aligned}$$

- Distribution is not known (or no distribution).
- Consequences are observed once.
- Risk level is high.
- Example: Disaster management.

Sequential decision-making under uncertainty



In optimization under uncertainty the timing of decisions is important.

The difficulty of solution can increase with the number of decision stages.