Artificial evolution and applications

Pierrick Legrand March 16, 2022

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PART 1: Artificial Evolution

PART 2: Applications

- Evolutionary computation for EEG classification
- Regularity estimation with Genetic Programming

Darwinism 0000	Artificial Darwinism	Genetic Algorithms	Evolution Strategies	Genetic Programming

PART 1: Artificial Evolution

Darwinism	Artificial Darwinism	Genetic Algorithms	Evolution Strategies	Genetic Programming

Darwinism

Artificial Darwinism

A set of techniques grouped under a generic term Ingredients Evolutionary loop Example

Genetic Algorithms

Discrete representation: Genetic Algorithms

Evolution strategies

Continuous representation: Evolution Strategies

Genetic Programming

Functional representation: Genetic programming Example: Using GP for regression

Darwinism ●୦୦୦	Artificial Darwinism	Genetic Algorithms	Evolution Strategies	Genetic Programming

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0000				

Darwinism, evolutionism

Charles Robert Darwin (1809-1882). 1831 - 5 years on the HMS Beagle to Galapagos Islands. November 1859, book "On the origin of species".

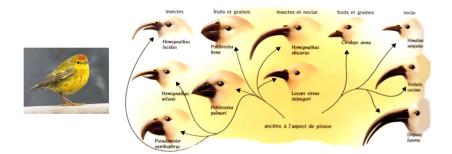




Darwinism ○○●○	Artificial Darwinism	Genetic Algorithms	Evolution Strategies	Genetic Programming

The Darwin Finch

"We can say, by metaphor, that natural selection seeks, at every moment and worldwide, the slightest variations; it repels those that are harmful, it preserves and accumulates those that are useful; she works in silence, imperceptibly, everywhere and always, as soon as the opportunity arises, to improve all organized beings relative to their organic and inorganic living conditions" (Darwin, 1859).



Darwinism ୦୦୦●	Artificial Darwinism	Genetic Algorithms	Evolution Strategies	Genetic Programming
Simple meca	anisms			

- 1 Variations, macroscopic and microscopic, within species.
- 2 Fight for survival.
- 3 Natural selection: triumph of the lineage that has a useful variation in its environment.

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Darwinism

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Artificial Darwinism

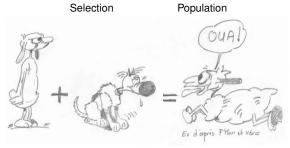
Stochastic optimization which uses mechanisms inspired by the biological evolution, such as:

- reproduction,
- mutation,
- selection and
- survival of the strongest individuals

Darwinism	Artificial Darwinism	Genetic Algorithms	Evolution Strategies	Genetic Programming
	00000			
A set of technic	ques grouped under a gene	ric term		

	Genetic Algorithms (GA)
Evolutionary Algorithms	Evolution Strategies (ES)
	Genetic Programming (GP)

Darwinism 0000	Artificial Darwinism	Genetic Algorithms	Evolution Strategies	Genetic Programming
Ingredients				
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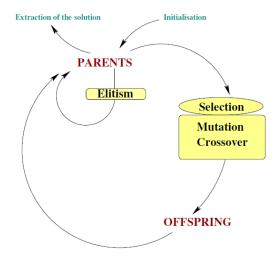


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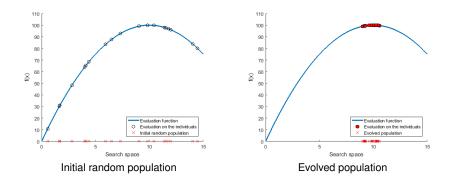
Genetic Operators

Darwinism 0000	Artificial Darwinism	Genetic Algorithms	Evolution Strategies	Genetic Programming
Evolutionary lo	ор			



Darwinism	Artificial Darwinism	Genetic Algorithms	Evolution Strategies	Genetic Programming
Example				

Potential Solutions = Individuals in a population



Darwinism 0000	Artificial Darwinism	Genetic Algorithms ●○	Evolution Strategies	Genetic Programming

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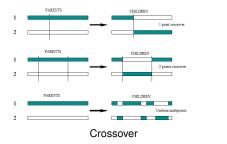
Genetic Programming

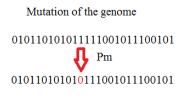
Functional representation: Genetic programming Example: Using GP for regression

Darwinism	Artificial Darwinism	Genetic Algorithms ○●	Evolution Strategies	Genetic Programming	
Discrete representation: Genetic Algorithms					

Each individual is represented by a binary string.

John H. Holland (1960, 1975), David Goldberg (1989)





Mutation

Darwinism	Artificial Darwinism	Genetic Algorithms	Evolution Strategies ●○	Genetic Programming

Darwinism

Artificial Darwinism

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Darwinism 0000	Artificial Darwinism	Genetic Algorithms	Evolution Strategies ○●	Genetic Programming	
Continuous representation: Evolution Strategies					

Each individual is a vector in \mathbb{R}^n .

Hans-Paul Schwefel (1970)

Barycentric crossover

 $\begin{aligned} \forall i \in \{1,..,n\}, x_i^{children} &= \alpha x_i^{father} + (1-\alpha) x_i^{mother} \\ \alpha \text{ random value in } [-\epsilon,1+\epsilon]. \end{aligned}$

Gaussian mutation

 $\forall i \in \{1,..,n\}, x_i^{children} = x_i^{children} + N(0,\sigma)$ Two parameters P_m and σ .

Darwinism 0000	Artificial Darwinism	Genetic Algorithms	Evolution Strategies ○●	Genetic Programming	
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Demo

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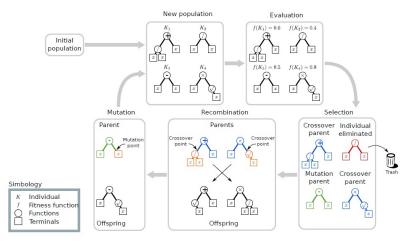
Genetic Programming

Functional representation: Genetic programming Example: Using GP for regression

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Functional representation: Genetic programming				

Definition

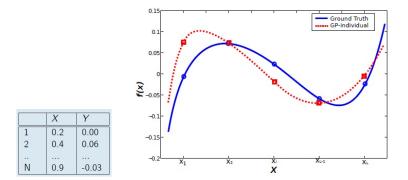
Genetic programming (GP) is an evolutionary computation (EC) technique that automatically solves problems without requiring the user to know or specify the form or structure of the solution in advance [Koza, 1992].



Darwinism	Artificial Darwinism	Genetic Algorithms	Evolution Strategies	Genetic Programming ○○●
Example: Using	g GP for regression			

Symbolic Regression

Given a set of input data X and a set of desired outputs Y, find a function f such that: $f(X_i) = Y_i \quad \forall i \in \{1, N\}$



PART 2: Applications

Evolutionary computation for EEG classification

This work is related to the PhD thesis of Laurent Vezard and developed in the context of the PSI Region Project and the ACOBSEC European project. A slightly different version has been published in a book chapter. Eduardo Miranda; Julien Castet; Benjamin Knapp. Guide to Brain-Computer Music Interfacing, Springer, 2014. Work carried out with Laurent Vézard, Marie Chavent, Frédérique Faïta-Aïnseba. EEG data Acquisition Acquisition Protocole

Feature Extraction Slope Criterion

Evolutionary Algorithm Design Results

Goal

- Characterize the state of alertness of a person from his electroencephalogram (EEG).

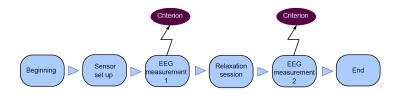
EEG data Acquisition Acquisition Protocole

Feature Extraction Slope Criterion

Evolutionary Algorithm Design Results

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- First EEG recording: subject in a normal state of alertness: "normal"
- Second EEG recording: subject in a state of low vigilance: "relax"



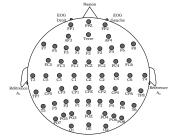
EEG data Acquisition

Acquisition Protocole

Feature Extraction

Evolutionary Algorithm





- EEG headset installation time: 45 minuts.
- Subject with open eyes.
- Sampling frequency: 256Hz.
- Recording time: 3 minuts (46000 sample points).

Campaigns:

Acquisition Protocole

- 58 electrodes renumbered from 1 to 58
- Subjects under 35, right-handed and non-smoker
- 58 subjects \Rightarrow 16 preserved

Relaxation session

20 minutes with a recorded voice offering 3 exercises:

- Autogenic training [Schultz1958]: repetition of sentences, self-hypnosis.
- Progressive muscle relaxation [Jacobson1974].
- Mental visualization (familiar places, smells, noises).

Acquisition Protocole

Feature Extraction

Evolutionary Algorithm

3 minutes of EEG recording before relaxation.

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Feature Extraction

Acquisition Protocole

3 minutes of EEG recording after relaxation.

Alpha waves

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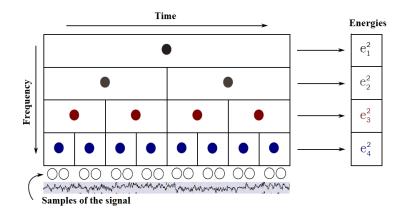
EEG data Acquisition Acquisition Protocole

Feature Extraction Slope Criterion

Evolutionary Algorithm Design Results

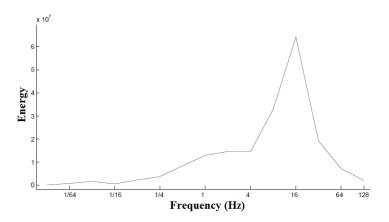
EEG data Acquisition	Feature Extraction ○●○○○○	Evolutionary Algorithm
Slope Criterion		

The dyadic grid gives a spatio-frequential representation of the discrete dyadic wavelet decomposition



EEG data Acquisition	Feature Extraction ○○●○○○	Evolutionary Algorithm
Slope Criterion		

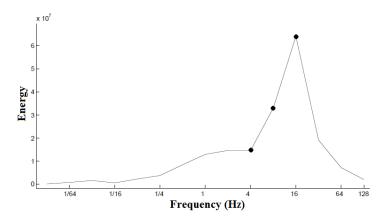
- Alpha: 8 12Hz.
- Waves characteristics of a relaxed state.



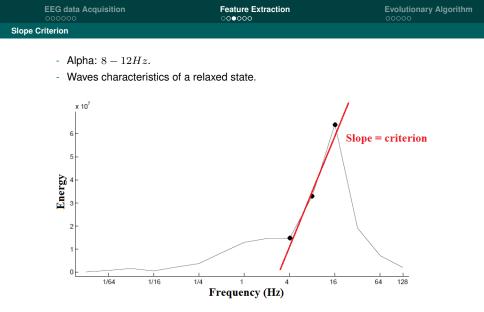
Linear regression between 4 and 16Hz.

EEG data Acquisition	Feature Extraction ○○●○○○	Evolutionary Algorithm
Slope Criterion		

- Alpha: 8 12Hz.
- Waves characteristics of a relaxed state.



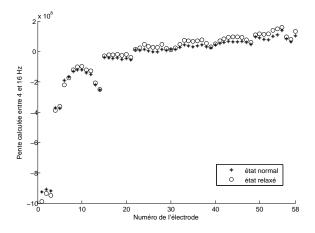
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Linear regression between 4 and 16Hz.

Evolutionary Algorithm

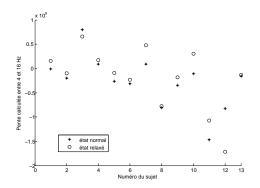
Slope Criterion



Slope criterion, sum on subjects for each electrode

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Slope Criterion



Slope criterion, sum on electrodes for each subjects

- ⇒ Very strong inter-subject variability
- ⇒ This criterion that does not allow to build a powerful classifier for different subjects.

EEG (lata	Acquisition

Slope Criterion

Classification: Usual methods

	K nearest	Binary	Random	Discriminant	Sparse Discriminant
	neighbors	decision trees	forests	PLS	PLS
Mean	37.28	33.98	32.03	40.63	36.25
Standard Deviation	10.47	5.15	6.46	8.55	7.96

Mean and standard deviations of Correct Classification Rates for different classification methods applied on slope criterion.

This approach is not efficient

Our contribution: Design a relevant **evolutionary algorithm** to solve this task of classification.

 \Rightarrow Find the relevant electrodes.

 \Rightarrow Find the relevant frequencies for the calculation of the slope criterion.

EEG data Acquisition Acquisition Protocole

Feature Extraction Slope Criterion

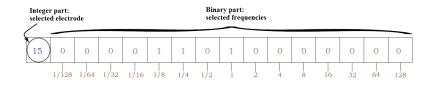
Evolutionary Algorithm Design Results

EEG data	Acquisition

Evolutionary Algorithm

Design

Example of a genome in the evolutionary algorithm

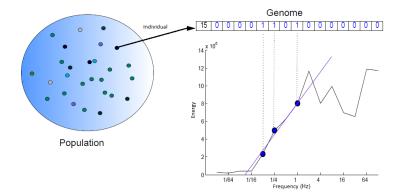


EEG	data	Acq	uisition

Evolutionary Algorithm

Design

Relationship between the genome and the calculation of the slope criterion



EEG data	Acquisition

Results

Average correct classification rate

Evaluation	CCR		
Method	Mean	Standard deviation	
CART	86.68	1.87	
SVC	83.49	2.37	

Average and standard deviations of the correct classification rates obtained for the 100 runs of the evolutionary algorithm and for two methods of evaluation.

Results

Best genome

Evaluation		BEST genome	
method	Selected	Selected	Correct classification
	electrode	frequency (Hz)	rates
CART	F4	1/8, 1/4, 2, 4 et 64	89,33%
SVC	F2	1/32, 1/16, 2, 4, 8, 64 et 128	89,33%

Table summarizing the two best genomes found during the 100 runs of the genetic algorithm with two methods of evaluation.

Regularity estimation with Genetic Programming

Joint work with Leonardo Trujillo, Gustavo Olague and Jacques Levy-Vehel. Evolving estimators of the pointwise Hölder exponent with Genetic Programming. Information Sciences 209 (Nov. 2012), 61-79. Hölderian Regularity

Contribution Training set

Results

Hölderian Regularity

Contribution Training set

Results

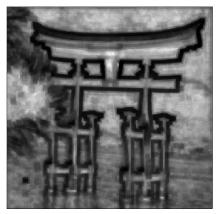
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Contribution	

Hölder exponent

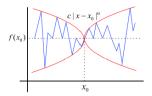
Mathematical tool that measures the regularity of a signal around each point.

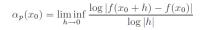




Hölderian Regularity ○○●	Contribution	Results

General motivation



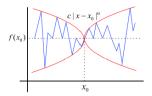


Hölderian envelope of signal f at point x_0

• For real-world signals the Hölder exponent must be estimated for each point.

Hölderian Regularity	Contribution	Results
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General motivation



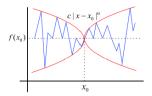


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- Several estimation methods exist, but most methods are slow or highly parameterized;

Hölderian Regularity ○○●	Contribution	Results
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General motivation





Hölderian envelope of signal f at point x_0

- For real-world signals the Hölder exponent must be estimated for each point.
- Several estimation methods exist, but most methods are slow or highly parameterized;
- Therefore there use is not common (particularly in applications where speed can be of importance)

Hölderian Regularity

Contribution Training set

Results

• Evolve estimators of the pointwise Hölder exponent for 2D signals with Genetic Programming.

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- GP evolves estimators that are **accurate** and **fast**.
- · Evolution is a one-shot process, evolved estimators can be used easily.

Awards

- Best Paper Award in the track Genetic Programming, GECCO 2010, Portland, Oregon.
- Humies Award Finalist, GECCO 2013, Amsterdam, The Netherland.



We generate three groups of images with **FracLab**, using three different functions that take as input the point coordinates (x, y) of an image and provide as output the desired regularity; these functions are:

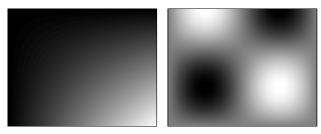
1 a Polynomial
$$p_1(x, y) = 0.1 + 0.8xy;$$

2 a Sine
$$p_2(x,y) = 0.5 + 0.2(sin(2\pi x))(cos(\frac{3}{2}\pi y));$$

3 an *Exponential*
$$p_3(x,y) = 0.3 + \frac{0.3}{1+e^{-100(x-0.7)}}$$
.

These functions provide the prescribed regularity needed to build the synthetic images used for training and testing of our evolved operators.

Training set



(a) Polynomial p_1



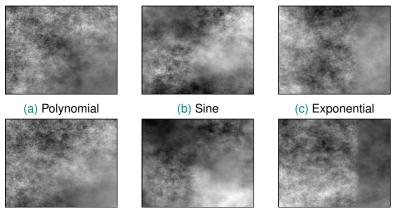


(c) Exponential p_3

Pierrick Legrand IMB/INRIA/UBX

Training set

Contribution ○○○○●



(d) Polynomial

(e) Sine

(f) Exponential

These images have a prescribed regularity given by functions p_1 (Polynomial), p_2 (Sine) and p_3 (Exponential).

Hölderian Regularity

Contribution Training set

Results

Results ○●

Results: Real Images



Original Image



GP-Estimator



Traditional Method



GP-Estimator