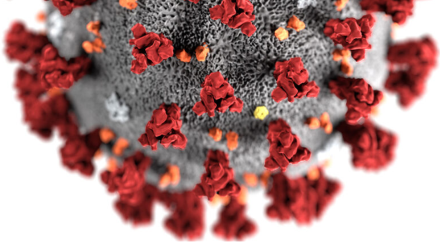


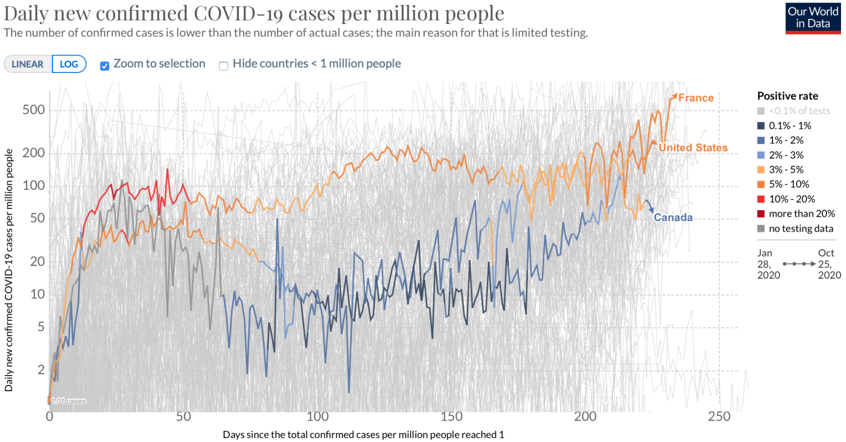
Epidemiology and Economics of Physical Distancing During Infectious Disease Outbreaks

Troy Day
Department of Mathematics and Statistics

28 October 2020

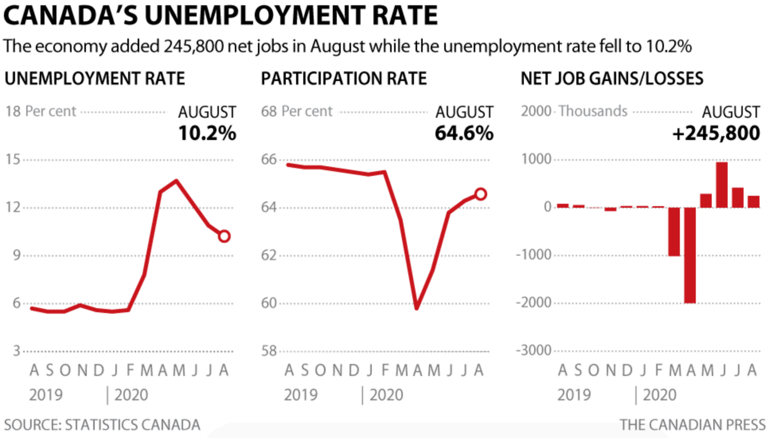
Queen's University

New Cases



Daily new confirmed COVID-19 cases per million people
The number of confirmed cases is lower than the number of actual cases; the main reason for that is limited testing.
Our World in Data
LINEAR
LOG
Zoom to selection
Hide countries < 1 million people
Daily new confirmed COVID-19 cases per million people
500
200
100
50
20
10
5
2
1.01 cases
0
50
100
150
200
250
Days since the total confirmed cases per million people reached 1
France
United States
Canada
Positive rate
<0.1% of tests
0.1% - 1%
1% - 2%
2% - 3%
3% - 5%
5% - 10%
10% - 20%
more than 20%
no testing data
Jan 28, 2020
Oct 25, 2020

Job Losses


CANADA'S UNEMPLOYMENT RATE
The economy added 245,800 net jobs in August while the unemployment rate fell to 10.2%
UNEMPLOYMENT RATE
18 Per cent
AUGUST
10.2%
15
12
9
6
3
A S O N D J F M A M J J A
2019
2020
PARTICIPATION RATE
68 Per cent
AUGUST
64.6%
66
64
62
60
58
A S O N D J F M A M J J A
2019
2020
NET JOB GAINS/LOSSES
2000 Thousands
AUGUST
+245,800
1000
0
-1000
-2000
-3000
A S O N D J F M A M J J A
2019
2020
SOURCE: STATISTICS CANADA
THE CANADIAN PRESS

Outline

1 Background

2 Setup

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Outline

1 **Background**

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Physical Distancing is a Game

For example:

Epidemiologically

- If everyone else is staying away from the office then there is little risk if you go into work. But if everyone follows this reasoning then problems can clearly arise.

Economically

- If you run a business then there is strong incentive to stay open if customer are out. But this incentive is much weaker if most people are physical distancing.

Some Canonical Games

Trivial Game

Opponent's Behaviour

Your Behaviour	Home	Work
Home	0	1
Work	1	2

Coordination Game

Opponent's Behaviour

Your Behaviour	Home	Work
Home	3	1
Work	1	2

Diversification Game

Opponent's Behaviour

Your Behaviour	Home	Work
Home	0	1
Work	1	0

Prisoner's Dilemma Game

Opponent's Behaviour

Your Behaviour	Home	Work
Home	3	1
Work	4	2

Overall Model Structure

- A population of citizens or ‘agents’ who each act individually in their own personal interest (e.g., physical distancing or not).
- The population is large enough that any single agent makes up a negligibly small component.
- A government leader who can compel coordinated action among agents if this would be to their benefit (e.g., stay-at-home orders or back-to-work orders)

Questions:

- How do an individual’s incentives change over the course of a disease outbreak?
- How should a government leader intervene?

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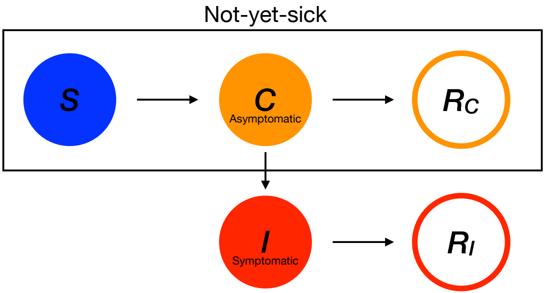
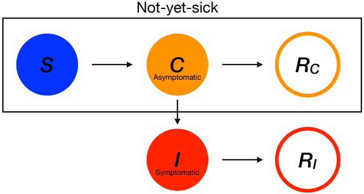
The Epidemiology

- States $\{\mathcal{S}, \mathcal{C}, \mathcal{I}, \mathcal{R}_\mathcal{C}, \mathcal{R}_\mathcal{I}\}$
- Fraction of population in each state denoted by S, C, I, R_C, R_I
- Recovered individuals in states $\mathcal{R}_\mathcal{C}$ and $\mathcal{R}_\mathcal{I}$ are immune.

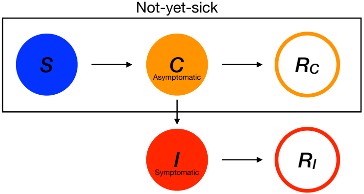


The Decision Variable

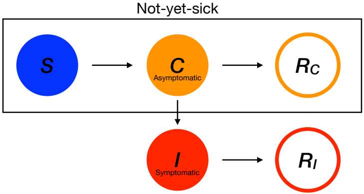


- $\delta_i(t) \in \{0, 1\}$ is distancing strategy for an agent in state i
- $\alpha \in (0, 1)$ is effectiveness of distancing (if $\delta_i(t) = 1$ then contact rate is proportional to $1 - \alpha$)
- $d_i(t) = \mathbb{E}[\delta_i(t)]$ is frequency of state i agents with $\delta_i(t) = 1$
- Contact rate of class of state i individuals at time t is proportional to $d_i(t) \times (1 - \alpha) + (1 - d_i(t)) \times 1 = 1 - \alpha d_i(t)$

Information States & Assumptions



- All symptomatic infections distance ($\delta_{\mathcal{I}}(t) \equiv 1$, $d_{\mathcal{I}}(t) \equiv 1$)
- Those recovered from symptomatic infection do not distance ($\delta_{\mathcal{R}_{\mathcal{I}}}(t) \equiv 0$, $d_{\mathcal{R}_{\mathcal{I}}}(t) \equiv 0$)
- States $\{\mathcal{S}, \mathcal{C}, \mathcal{R}_{\mathcal{C}}\}$ form a single information state, $\mathcal{N}$, for 'not-yet-sick' ($\delta_{\mathcal{S}} = \delta_{\mathcal{C}} = \delta_{\mathcal{R}_{\mathcal{C}}} = \delta_{\mathcal{N}}$)



$$
\begin{aligned}
S' &= -\lambda(C, I, d_{\mathcal{N}})(1 - \alpha d_{\mathcal{N}})S \\
C' &= \lambda(C, I, d_{\mathcal{N}})(1 - \alpha d_{\mathcal{N}})S - \sigma C - \gamma C \\
I' &= \sigma C - \gamma I - vI \\
R_C' &= \gamma C \\
R_I' &= \gamma I
\end{aligned}
$$

$$\lambda(C, I, d_{\mathcal{N}}) = \beta_C(1 - \alpha d_{\mathcal{N}})C + \beta_I(1 - \alpha)I$$

$$S(0) = 1 - C(0) - I(0)\ ,\ C(0) \approx 0,\ I(0) \approx 0,\ R_C(0) = 0,\ R_I(0) = 0$$

Individual State Transitions

Let $Y(t) \in \{\mathcal{S}, \mathcal{C}, \mathcal{I}, \mathcal{R}_\mathcal{C}, \mathcal{R}_\mathcal{I}\}$ be a stochastic process (CTMC) representing the state of an individual at time t. The probability, $p_i(t)$, that $Y(t)$ is in state i at time t is governed by:

$$
\begin{aligned}
p'_\mathcal{S} &= -\lambda(C, I, d_\mathcal{N})(1 - \alpha\delta_\mathcal{N})p_\mathcal{S} \\
p'_\mathcal{C} &= \lambda(C, I, d_\mathcal{N})(1 - \alpha\delta_\mathcal{N})p_\mathcal{S} - \sigma p_\mathcal{C} - \gamma p_\mathcal{C} \\
p'_\mathcal{I} &= \sigma p_\mathcal{C} - \gamma p_\mathcal{I} - v p_\mathcal{I} \\
p'_{\mathcal{R}_\mathcal{C}} &= \gamma p_\mathcal{C} \\
p'_{\mathcal{R}_\mathcal{I}} &= \gamma p_\mathcal{I}
\end{aligned}
$$

Initial conditions: $p_\mathcal{S}(0)$, $p_\mathcal{C}(0)$, $p_\mathcal{I}(0)$, $p_{\mathcal{R}_\mathcal{C}}(0)$, $p_{\mathcal{R}_\mathcal{I}}(0)$

The Epidemiology

Population Dynamics: $\dot{x} = g(x)$

$$x = \begin{pmatrix} S \\ C \\ I \\ R_C \\ R_I \end{pmatrix} \qquad g(x) = \begin{pmatrix} -\lambda(C, I, d_{\mathcal{N}})(1-\alpha d_{\mathcal{N}})S \\ \lambda(C, I, d_{\mathcal{N}})(1-\alpha d_{\mathcal{N}})S - \sigma C - \gamma C \\ \sigma C - \gamma I - vI \\ \gamma C \\ \gamma I \end{pmatrix}$$

Individual State Dynamics: $\dot{p} = Q(x) \cdot p$

$$p = \begin{pmatrix} p_{\mathcal{S}} \\ p_{\mathcal{C}} \\ p_{\mathcal{I}} \\ p_{\mathcal{R}_\mathcal{C}} \\ p_{\mathcal{R}_\mathcal{I}} \end{pmatrix} \quad Q(x) = \begin{pmatrix} -\lambda(C, I, d_{\mathcal{N}})(1-\alpha d_{\mathcal{N}}) & 0 & 0 & 0 & 0 \\ \lambda(C, I, d_{\mathcal{N}})(1-\alpha d_{\mathcal{N}}) & -\sigma-\gamma & 0 & 0 & 0 \\ 0 & -\sigma & -v-\gamma & 0 & 0 \\ 0 & \gamma & 0 & 0 & 0 \\ 0 & 0 & \gamma & 0 & 0 \end{pmatrix}$$

The Economy McAdams, D. 2020. Covid Economics: 16:115-134

The rate of economic flow to an individual agent at time t is

$$b(\delta_i; A(t)) = a_0 + (1 - \alpha\delta_i)F(A)$$

where $A(t) = (1 - \alpha d_{\mathcal{N}}(t))N(t) + (1 - \alpha)I(t) + R_I(t)$ is the average availability of others for interactions and $a_0 > 0$.

- $F(A)$ represents the economic benefit of activity to an individual agent, which depends on the activity of others (we assume $F(0) > 0, F'(A) > 0$)
- We will typically take $F(A) = a_1 + a_2 A$ with $a_1 > 0$ and $a_2 > 0$.

The Objective

Define:

- $p_{i|j}(\tau;t) = \mathbb{P}(Y(\tau) = i | Y(t) = j)$ with $\tau \geq t$
- h is discounting rate (including other mortality, development of treatments, etc.)
- $V_i(x(t))$ as present value of all economic flow from time t onward for an agent in state i

$$V_{\mathcal{S}}(x(t)) = \int_t^\infty e^{-h(\tau-t)} \left((p_{\mathcal{S}|\mathcal{S}} + p_{\mathcal{C}|\mathcal{S}} + p_{\mathcal{R}_\mathcal{C}|\mathcal{S}})b(\delta_\mathcal{N}; A) + p_{\mathcal{I}|\mathcal{S}} b(1;A) + p_{\mathcal{R}_\mathcal{I}|\mathcal{S}} b(0;A)\right) d\tau$$

$$V_{\mathcal{C}}(x(t)) = \int_t^\infty e^{-h(\tau-t)} \left((p_{\mathcal{C}|\mathcal{C}} + p_{\mathcal{R}_\mathcal{C}|\mathcal{C}})b(\delta_\mathcal{N}; A) + p_{\mathcal{I}|\mathcal{C}} b(1;A) + p_{\mathcal{R}_\mathcal{I}|\mathcal{C}} b(0;A)\right) d\tau$$

$$V_{\mathcal{I}}(x(t)) = \int_t^\infty e^{-h(\tau-t)} \left(p_{\mathcal{I}|\mathcal{I}} b(1;A) + p_{\mathcal{R}_\mathcal{I}|\mathcal{I}} b(0;A)\right) d\tau$$

$$V_{\mathcal{R}_\mathcal{C}}(x(t)) = \int_t^\infty e^{-h(\tau-t)} b(\delta_\mathcal{N}; A) d\tau$$

$$V_{\mathcal{R}_\mathcal{I}}(x(t)) = \int_t^\infty e^{-h(\tau-t)} b(0; A) d\tau$$

Maximize: $V_\mathcal{N}(x(t)) = \frac{p_\mathcal{S}}{p_\mathcal{N}} V_\mathcal{S}(x(t)) + \frac{p_\mathcal{C}}{p_\mathcal{N}} V_\mathcal{C}(x(t)) + \frac{p_{\mathcal{R}_\mathcal{C}}}{p_\mathcal{N}} V_{\mathcal{R}_\mathcal{C}}(x(t))$

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$$V_i(x(t)) = \int_t^\infty e^{-h(\tau-t)} \left((p_{\mathcal{S}|i} + p_{\mathcal{C}|i} + p_{\mathcal{R}_\mathcal{C}|i}) b(\delta_\mathcal{N}; A) + p_{\mathcal{I}|i} b(1; A) + p_{\mathcal{R}_\mathcal{I}|i} b(0; A)\right) d\tau$$

Consider a single agent's decision in small interval of time from t to $t + dt$, assuming all agents' strategies from $t + dt$ onward are optimal (denoted by a *).

$$\begin{aligned} V_i(x(t)) &\approx b(\delta_i; A) dt \\ &+ \int_{t+dt}^\infty e^{-h(\tau-t)} \left((p_{\mathcal{S}|i} + p_{\mathcal{C}|i} + p_{\mathcal{R}_\mathcal{C}|i}) b(\delta^*_\mathcal{N}; A) + p_{\mathcal{I}|i} b(1; A) + p_{\mathcal{R}_\mathcal{I}|i} b(0; A)\right) d\tau \\ &\approx b(\delta_i; A) dt + e^{-hdt} \sum_k \mathbb{P}(Y(t+dt) = k | Y(t) = i) V_k^*(x(t+dt)) \\ &\approx b(\delta_i; A) dt + V_i^*(x(t)) - hV_i^*(x(t)) dt \\ &+ \sum_k \dot{\mathbb{P}}(Y(t) = k | Y(t) = i) V_k^*(x(t)) dt + \dot{V}_i^*(x(t)) dt \end{aligned}$$

Hamilton-Jacobi-Bellman Equation

$$h\vec{V} = \vec{b} + Q(x)^T \cdot \vec{V} + J \cdot g$$

$$\vec{V} = \begin{pmatrix} V_{\mathcal{S}} \\ V_{\mathcal{C}} \\ V_{\mathcal{I}} \\ V_{\mathcal{R}_\mathcal{C}} \\ V_{\mathcal{R}_\mathcal{I}} \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} b(\delta_{\mathcal{N}}; A) \\ b(\delta_{\mathcal{N}}; A) \\ b(1; A) \\ b(\delta_{\mathcal{N}}; A) \\ b(0; A) \end{pmatrix} \qquad J = \text{Jacobian of } \vec{V} \text{ wrt } x(t)$$

Maximize: $V_{\mathcal{N}}(x(t)) = \frac{p_{\mathcal{S}}}{p_{\mathcal{N}}} V_{\mathcal{S}}(x(t)) + \frac{p_{\mathcal{C}}}{p_{\mathcal{N}}} V_{\mathcal{C}}(x(t)) + \frac{p_{\mathcal{R}_\mathcal{C}}}{p_{\mathcal{N}}} V_{\mathcal{R}_\mathcal{C}}(x(t))$

Agent's Objective Function

Instantaneous Reward:

$$\pi(\delta_{\mathcal{N}}, d_{\mathcal{N}}) =$$

$$\underbrace{a_0}_{\text{baseline reward}} + \underbrace{(1-\alpha\delta_{\mathcal{N}})}_{\text{agent's activity level}} \underbrace{[F(A_{d_{\mathcal{N}}}) - q_{\mathcal{S}}\lambda_{d_{\mathcal{N}}} H(t)]}_{\text{net payoff for activity}} + \underbrace{(1-\alpha d_{\mathcal{N}}) S \lambda_{d_{\mathcal{N}}} D(t)}_{\text{payoff to agent of others' actions}}$$

where $H(t) = V^*_{\mathcal{S}} - V^*_{\mathcal{C}}$, $D(t) = \left(\frac{\partial V^*_{\mathcal{N}}}{\partial C} - \frac{\partial V^*_{\mathcal{N}}}{\partial S}\right)$, $q_{\mathcal{S}} = p_{\mathcal{S}}/p_{\mathcal{N}}$

$H(t) =$ cost to an $\mathcal{N}$-agent of getting infected at time t
$D(t) =$ payoff to an $\mathcal{N}$-agent of others getting infected at time t

Game Definitions

- No-distancing, trivial game
  $\pi(0, d_{\mathcal{N}}) > \pi(1, d_{\mathcal{N}})$ and $\pi(0,0) > \pi(1,1)$
- Distancing, trivial game
  $\pi(0, d_{\mathcal{N}}) < \pi(1, d_{\mathcal{N}})$ and $\pi(0,0) < \pi(1,1)$
- No-distancing, prisoner's dilemma
  $\pi(0, d_{\mathcal{N}}) > \pi(1, d_{\mathcal{N}})$ and $\pi(0,0) < \pi(1,1)$
- Distancing, prisoner's dilemma
  $\pi(0, d_{\mathcal{N}}) < \pi(1, d_{\mathcal{N}})$ and $\pi(0,0) > \pi(1,1)$
- Coordination game
  $\pi(0,0) > \pi(1,0)$ and $\pi(0,1) < \pi(1,1)$
- Diversification game
  $\pi(0,0) < \pi(1,0)$ and $\pi(0,1) > \pi(1,1)$

Suppose both $C(0) > 0$ and $I(0) > 0$ are arbitrarily small.

Lemma (The outbreak is transient)

$C \to 0$ and $I \to 0$ as $t \to \infty$, regardless of agents' behaviours.

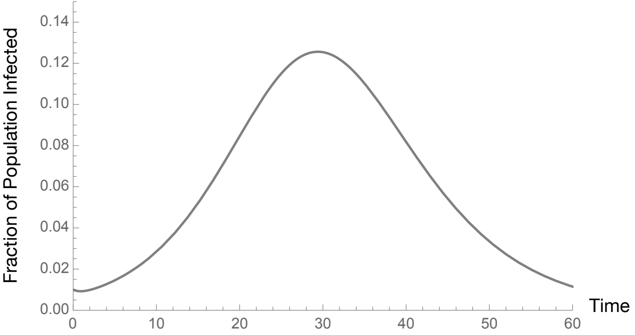
Lemma (Nobody should distance before or after the outbreak)

In the limits $t \to 0$ and $t \to \infty$, agents are engaged in a no-distancing, trivial game.

Theorem (Asymptomatic Transmission)

Part (A). Suppose $\beta_I = 0$. If the outbreak grows large enough to incentivize agents to distance, then they become engaged in a ***diversification game*** *at this point. Likewise, as the outbreak subsides and $I \to 0$ and $C \to 0$, agents will again become engaged in a* ***diversification game*** *before all agents cease distancing. Furthermore, if the outbreak is large enough, then there is a period of time during the middle of the outbreak where all agents are engaged in a* ***distancing, trivial game*** *(i.e., they all prefer to distance).*

Part (B). Suppose further that the payoff for others becoming infected, $D(t)$, is always non-positive. Then agents will become engaged in a ***no-distancing, prisoner's dilemma*** *immediately prior to, and immediately after, they engage in the diversification game. At such times, all agents would benefit if a government leader intervened and enforced a stay-at-home order.*

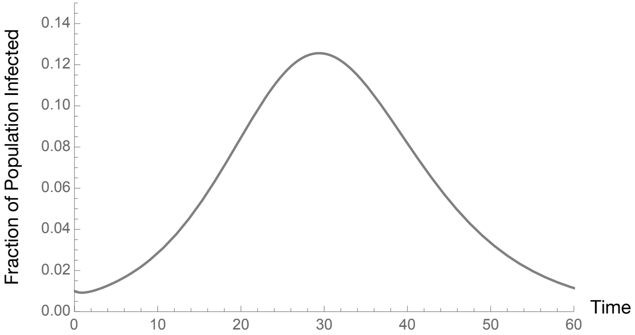


Asymptomatic (SARS-CoV-2):

Theorem (Symptomatic Transmission)

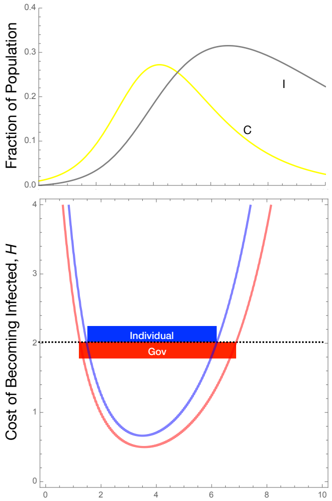
Part (A). Suppose $\beta_C = 0$. If the outbreak grows large enough to incentivize agents to distance, then they become engaged in a ***coordination game****. Likewise, as the outbreak subsides and $I \to 0$ and $C \to 0$, agents will again become engaged in a* ***coordination game*** *before all agents cease distancing. Furthermore, if the outbreak is large enough, then there is a period of time during the middle of the outbreak where all agents are engaged in a* ***distancing, trivial game*** *(i.e., they all prefer to distance).*

Part (B). Suppose further that the payoff for others becoming infected, $D(t)$, is always non-negative. Then agents will become engaged in a ***distancing, prisoner's dilemma*** *immediately prior to, and immediately after, the distancing, trivial game. At such times, all agents would benefit if a government leader intervened and enforced a back-to-work order.*

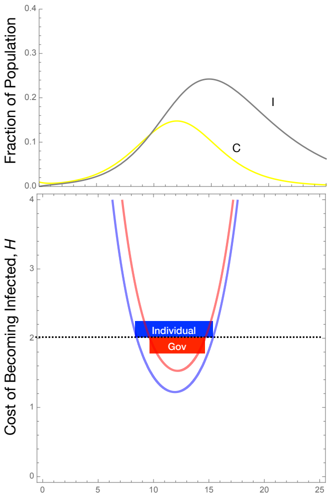


Symptomatic
(SARS-CoV):

Asymptomatic Transmission



Symptomatic Transmission



$$\lambda_{d_\mathcal{N}} = \beta_C(1-\alpha d_\mathcal{N})C + \beta_I(1-\alpha)I$$

Asymptomatic (SARS-CoV-2)

$$\lambda_{d_\mathcal{N}} = (1-\alpha d_\mathcal{N})\lambda_0$$

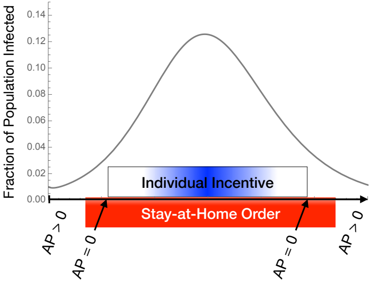
Collective Distancing *Increases* Activity Payoff (AP)

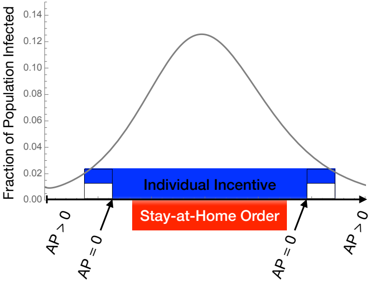
Symptomatic (SARS-CoV)

$$\lambda_{d_\mathcal{N}} = \lambda_0$$

Collective Distancing *Decreases* Activity Payoff (AP)

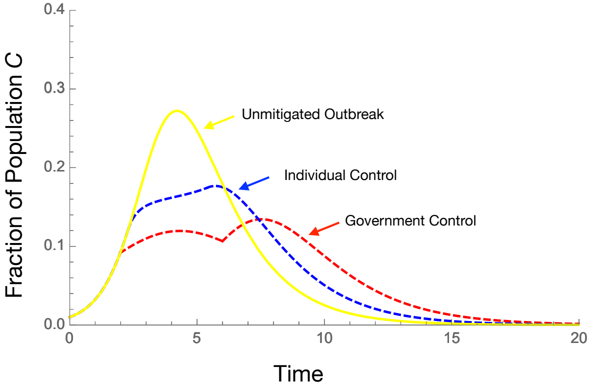
$$\pi(\delta_\mathcal{N}, d_\mathcal{N}) = a_0 + (1-\alpha\delta_\mathcal{N})\underbrace{\left[F(A_{d_\mathcal{N}}) - q_\mathcal{S}\lambda_{d_\mathcal{N}}H\right]}_{\text{Activity Payoff}}$$





Acting on Incentives Changes the Game

Asymptomatic Transmission:



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Summary

1. Asymptomatic Transmission: Collective distancing increases activity payoff.

Asymptomatic (SARS-CoV-2):

2. Symptomatic Transmission: Collective distancing decreases activity payoff.

Symptomatic (SARS-CoV):

3. Acting on incentives (individuals or government) can change the nature of the game.

Joint work with:

David McAdams, Fuqua School of Business and Economics Department, Duke University

NSERC CRSNG People. Discovery. Innovation.

Previous Work

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