Numerical Analysis of a Integrator for Size-Structured Population Models in an Environment with Dynamical Food-Source

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ABSTRACT

We will study the numerical integration of a nonlinear model that describes the dynamics of size-structured populations feeding on a dynamical food-source. Such models are determined by the next equations:

$$u_t + (g(x, S(t))\, u)_x = -\mu(x, S(t))\, u, \quad 0 < x < x_M(t),\ t > 0, \tag{1}$$

$$g(0, S(t))\, u(0, t) = \int_0^{x_M(t)} \alpha(x, S(t))\, u(x, t)\, dx\ , \quad t > 0, \tag{2}$$

$$u(x, 0) = \phi(x), \quad 0 \le x \le x_M(t), \tag{3}$$

$$S'(t) = f(t, S(t), I(t)), \quad t > 0, \qquad S(0) = s_0, \tag{4}$$

$$I(t) = \int_0^{x_M(t)} \gamma(x, S(t))\, u(x, t)\, dx, \quad t > 0. \tag{5}$$

where $x_M(0) = x_M$ and $x_M(t)$ represents the characteristic curve that begins at the maximum size. The independent variables x and t represent, respectively, size and time, and the function $u(x, t)$ is the population density of individuals with size x at time t. The population dynamics is determined by the growth rate g, the mortality rate μ and the fertility rate α. In these models, the growth function g can change its sign. These vital functions depend on the structuring variable and on the available food resources that are given by the function $S(t)$. Also, the dynamics of such resources depends on to the distribution of the population such as the equations (4) and (5) reflect.

The present work carry out the numerical integration of equations (1)-(5) by means of a suitable method for the coupled partial integro-differential equation (that represents the dynamics of the population) and the ordinary differential equation (that gives the dynamics of the resource). We will analyse the consistency, stability and convergence properties of the numerical scheme. Special attention is devoted to the requirements imposed to the quadrature rule. Also, we will analyse the behaviour of such numerical scheme in the integration of the dynamics of *Daphnia magna*, feeding on a dynamical algal population [1,2].

Key Words: Size-structured models; Numerical Integration; Convergence

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References

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