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A chemostat model with general uptake and growth functions

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ABSTRACT

We investigate some properties of a very general model of growth in the chemostat. In the classical models of the chemostat, the function describing cellular growth is assumed to be a constant multiple of the function modeling substrate uptake. The constant of proportionality is called the *growth yield constant*. Here, this assumption of a constant describing growth yield is relaxed. Instead, we assume that the relationship between uptake and growth might depend on the substrate concentration and hence that the yield is variable.

We obtain criteria for the stability of equilibria and for the occurrence of a Hopf bifurcation. In particular, a Hopf bifurcation can occur if the uptake function is unimodal. Then, in this setting, we consider competition in the chemostat for a single substrate, in order to challenge the principle of competitive exclusion.

We consider two examples. In the first, the function describing the growth process is monotone and in the second it is unimodal. In both examples, in order to obtain a Hopf bifurcation, one of the competitors is assumed to have a variable yield, and its "uptake" is described by a unimodal function. However, the interpretation is different in each case. We provide a necessary condition for strong coexistence and a suffcient condition that guarantees the extinction of one or more species. We show numerically by means of bifurcation diagrams and simulations, that the competitive exclusion principle can be breached resulting in oscillatory coexistence of more than one species, that competitor-mediated coexistence is possible, and that these simple systems can have very complicated dynamics.

This is joint work with Sergei S. Pilyugin (University of Florida, Gainesville, Florida, USA) and Gail S.K. Wolkowicz (McMaster University, Hamilton, Ontario, Canada).