

Modelling hematopoiesis mediated by growth factors: Delay equations describing periodic hematological diseases

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ABSTRACT

Hematopoiesis is a complex biological process that leads to the production and regulation of blood cells [1]. It is based upon differentiation of stem cells under the action of growth factors. A mathematical approach of this process is proposed to carry out explanation on some blood diseases, characterized by oscillations in circulating blood cells [2]. A system of three differential equations with delay, corresponding to the cell cycle duration, is analyzed. The existence of a Hopf bifurcation for a positive steady-state is obtained through the study of an exponential polynomial characteristic equation with delay-dependent coefficients. Numerical simulations show that long period oscillations can be obtained in this model, corresponding to a destabilization of the feedback regulation between blood cells and growth factors. This stresses the localization of periodic hematological diseases in the feedback loop.

Key Words: delay differential equations, characteristic equation, delay-dependent coefficients, stability switch, Hopf bifurcation, cell population models, hematopoiesis, stem cells.

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