Semilinear parabolic boundary-value problem in bioreactors theory

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ABSTRACT

We consider a Plug Flow bioreactor with axial dispersion in which occurs a simple growth reaction (one biomass/one substrate). The dynamics of this bioreactor are described by a system of partial differential equations. This work is devoted to the analysis of this system: we aim at proving uniform boundedness of solutions and describing their omega-limit sets. To ease this analysis, we perform a linear change of state variables which transforms the system into two equations. One of them is nonlinear but the other one is linear. Thanks to the properties of the linear equation, we proved that the new system is equivalent to a nonautonomous semilinear parabolic equation of type

$$\begin{aligned} \frac{du}{dt} &= Au(t) + f(t,u), \\ u(0) &= u_0, \end{aligned} \tag{1}$$

where A is a linear operator in the Banach space $C[0,l]$ and (1) is asymptotically autonomous with limiting equation

$$\begin{aligned} \frac{du}{dt} &= Au(t) + g(u), \\ u(0) &= u_0 \end{aligned} \tag{2}$$

We prove a convergence result of solutions of (1) to steady state solutions of (2).

Key Words: Bioreactors, Semilinear equation, Asymptotically autonomous.
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