

## Asymptotic behavior of solutions to abstract logistic equations

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### ABSTRACT

We analyze the asymptotic behavior of solutions of the abstract differential equation  $u'(t) = Au(t) - F(u(t))u(t) + f$ . Our results are applicable to models of structured population dynamics in which the state space consists of population densities with respect to the structure variables. In the equation the linear term  $A$  corresponds to internal processes independent of crowding, the nonlinear logistic term  $F$  corresponds to the influence of crowding, and the source term  $f$  corresponds to external effects. We show that in the cases we analyze, the solutions stabilize in a way governed by the linear term. We illustrate the results with examples of models of structured population dynamics.

**Key Words:** abstract logistic equation, structured population, asymptotic behavior

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