

On the notion of invasion and the definition of invasion fitness

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ABSTRACT

A basic notion of adaptive dynamics is the *invasion fitness* $s_x(y)$ of a rare strategy y , given the resident strategies $x = (x_1, x_2, \dots, x_k)$. Traditionally it is defined as the long term average exponential growth rate of the strategy y (Metz *et al.* 1992). The rationale behind this notion is that it provides a convenient test for invasion: If $s_x(y) > 0$, then the strategy y can invade the resident population, whereas if $s_x(y) < 0$ it cannot invade and will eventually die out.

The definition of invasion fitness given by Metz *et al.* (1992) is very general in the sense that it applies to structured populations and the resident need not be at an ecological equilibrium. It has turned out to be successful in many applications. On the other hand, it has certain drawbacks that we shall point out and it is not applicable to all models that one encounters. It is the purpose of this talk to give a more general definition of invasion and invasion fitness that applies to a wider range of models. Our approach is based on the formalism of Diekmann *et al.* (1998, 2001) for physiologically structured populations.

Key Words: Adaptive dynamics, physiologically structured populations, environmental condition, time-dependent environments

AMS Classification: 92D15, 92D25

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