For which objective is birth process an optimal feedback in age structured population dynamics ?

Jacques Henry

INRIA Futurs - Bordeaux
Institut de Mathématiques - MAB
Université Bordeaux 1, 351, cours de la libération
33405 TALENCE cedex FRANCE
Jacques.Henry@inria.fr

ABSTRACT

We consider the usual linear model for the evolution of an age structured population.

$$u_t + u_a + \mu u = 0, \quad u(t,0) = v(t) \tag{1}$$

where $u(t,a)$ is the density of population with age a at time t, μ is the mortality rate. Usually the birth rate v(t) is given through a birth law with a fertility rate $\beta(a)$

$$v(t) = \int_0^{a_\dagger} \beta(a)u(a)da. \tag{2}$$

From the point of view of automatic control theory, this can be viewed as feedback mechanism. The purpose of the presentation is then to reformulate this "closed loop" as an open loop optimal control problem. In other words we consider the birth rate $v(t)$ as a control and we look for an objective function $J(v)$ such that corresponding optimal control in closed loop form gives exactly the law (2). We seek $J(v)$ of the form

$$J(v) = \frac{1}{2}\int_0^T \left(\int_0^{a_\dagger} c(a)u(t,a)\,da\right)^2 dt + \frac{1}{2}\int_0^T v(t)^2\,dt, \tag{3}$$

where the unknown is $c \in \mathcal{D}'([0,a_\dagger])$. For the homogeneous case the answer turns out to be

$$c(a) = \beta(a) - \delta. \tag{4}$$

The non homogeneous case with a right hand side f in (1) is also considered. Here a non homogeneous term has to be included to $J(v)$. We apply the method to control problems where a control v_1 is sought to minimize $J_1(u, v_1)$. This can be viewed as a control problem with respect to v and v_1 of minimization of first J and secondarily J_1.

AMS Classification: 92D25,49K20

References

[1] Aniţa, S. *Analysis and Control of Age-dependent Population Dynmics*, Kluwer Academic Publishers, 2000.
[2] Thieme, H. *Mathematics in Population Biology*, Princeton University Press, 2003.