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Discrete precipitations in a reaction-diffusion system

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Some reaction-diffusion systems exhibit a structure of spatially distinct bands of precipitated material, with clearly visible scaling properties. The phenomenon that we study has first been observed by Liesegang. In a spatial domain Ω an immobile reactant $\mathcal{B}$ is present, with uniform concentration b_0. Starting at an initial time $t = 0$, the boundary $\partial\Omega$ is brought in contact with a different reactant $\mathcal{A}$, that penetrates into Ω through a diffusive process. Inside Ω, $\mathcal{A}$ and $\mathcal{B}$ react, to produce a third substance $\mathcal{C}$. This substance on the one hand diffuses through Ω, and on the other hand crystallizes ("precipitates") to form an immobile product $\mathcal{D}$. This precipitation process starts as soon as the concentration c of $\mathcal{C}$ has surpassed a critical value $C_s > 0$; the precipitation rate is thought to linearly depend on c. In places where $\mathcal{D}$ has been formed, the precipitation process continues as long as c remains positive. We study here a one-dimensional reaction-diffusion system. Our purpose is to show that in a singular limit it gives rise to distinct regions where $\mathcal{D}$ is present, and that these regions have a regular spatial structure.