Permanence of structured population models and instability of the origin

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ABSTRACT

In this talk, we study the structured population models obeying ordinary differential equations, and consider the relationship between their permanence and instability of the origin. Ordinary differential equations for structured population models often take the form

$$\dot{\mathbf{x}} = A_{\mathbf{x}}\mathbf{x}, \tag{1}$$

where $\mathbf{x} = (x_1, x_2, \ldots, x_n)^\top$ and $A_{\mathbf{x}} = (a_{ij}(\mathbf{x}))$. We suppose that all individuals in this system belong to the same species, and that they are categorized into n classes by some property of the individuals (e.g., fecundity or habitat). Each x_i represents the population density in the i^{th} class (e.g., see [1,2] for examples of structured population models with the above form). We assume that (1) satisfies the following assumptions: (H0): The non-negative cone $\mathbb{R}^n_+$ is forward invariant; (H1): Each $a_{ij} : \mathbb{R}^n_+ \to \mathbb{R}$, $i, j \in \{1, \ldots, n\}$, is continuously differentiable; (H2): System (1) is dissipative. Under these assumptions, we provide a sufficient condition under which instability of the origin implies permanence of (1) in the following sense: there exists a $\delta > 0$ such that $\delta \leq \liminf_{t\to\infty} x_i(t) \leq \limsup_{t\to\infty} x_i(t) \leq 1/\delta$, $i = 1, 2, \ldots, n$, for all $\mathbf{x}(0) \in \mathbb{R}^n_+$ with $\mathbf{x}(0) \neq 0$, i.e., the population persists and class coexistence is achieved. Our results are applied to specific models including stage-structured and spatially structured population models. Furthermore, we show that our results are applicable to both stage-structured population models without irreducible life cycle graphs and spatially structured population models without irreducible patch connection graphs.

Key Words: Perron-Frobenius theorem, average Liapunov functions

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References

[1] Lu, Z. and Takeuchi, Y: Global asymptotic behavior in single-species discrete diffusion systems. *J. Math. Biol.* **32** (1993), 67–77.
[2] Thieme, H. R.: *Mathematics in population biology. Princeton Series in Theoretical and Computational Biology.* Princeton University Press, Princeton, NJ, 2003.