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Multiparametric bifurcation analysis of a basic two stage population model

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ABSTRACT

Population growth models that include age, stage or body size structure usually are rather sophisticated, involving partial or functional differential equations, difference equations, or integral equations. We will discuss the long-term dynamics of the most basic model, described in [1], for stage-structured populations, in which the per capita transition from the juvenile into the adult class is density dependent. The model is represented by a time autonomous, planar system of nonlinear differential equations for a single population. We find that the interaction of intra-adult competition and intra-juvenile competition gives rise to complex dynamics. A detailed numerical continuation study reveals a rich bifurcation structure for this two dimensional system, with co-existence of up to four equilibria. The organizing center is a degenerate Bogdanov-Takens (BT) codim-three point [2]. Two types of codim-two BT curves emanate from this point. From each point on these BT curves, in addition to a Hopf bifurcation codim-one curve, a codim-one homoclinic orbit originates. The periodic orbits originating from these homoclinics can be stable or unstable. We will study the type of degeneracy (which will appear to be elliptic) combining analytical (normal form analysis) and numerical continuation results. We expect that the dynamical structure of this relatively simple model is preserved or even enhanced in more complex stage-structured models.

Key Words: Bifurcation analysis - Bogdanov-Takens codim-three point - Elliptic sector - Homoclinic orbits to saddle, saddle-node and neutral saddle - Two-stage population model

AMS Classification: 34C23, 92D25, 37G05

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