International Workshop on Differential Equations in Mathematical Biology

Dynamics of similar populations

Géza Meszéna[1], Mats Gyllenberg[2], Frans J. Jacobs[3], Johan A. J. Metz[3,4]

[1]Department of Biological Physics, Eötvös University,
Pázmány 1A, H-1117 Budapest, Hungary
geza.meszena@elte.hu

[2]Rolf Nevanlinna Institute, Department of Mathematics and Statistics
FIN-00014 University of Helsinki, Finland
mats.gyllenberg@helsinki.fi

[3]Institute of Biology, Leiden University
P.O. Box 9516, NL-2300 RA Leiden, the Netherlands
jacobs@rulsfb.leidenuniv.nl

[4]Adaptive Dynamics Network, IIASA, A-2361 Laxenburg, Austria
metz@rulsfb.leidenuniv.nl

ABSTRACT

We study the joint population dynamics of *similar* populations in order to provide a robust underpinning for adaptive dynamics (AD) theory.

The continuity assumption behind AD is specified in a functional analytic framework. Population state is described by the Schwarcz distribution $\nu \in \mathcal{D}'(\mathcal{X})$ over the strategy space $\mathcal{X} \subset \mathbb{R}^k$. Dynamics is specified via the function $r : \mathcal{X} \times \mathcal{D}'(\mathcal{X}) \mapsto \mathbb{R}$; $r(y, \nu)$ giving the growth rate of strategy y when the strategy distribution is ν. We assume (functional) differentiability of r with respect to its second argument in a well-defined sense.

We restrict our attention to discrete distributions and assumes that the strategies in the support of the distribution are similar to each other. Their similarity implies that the relative dynamics of the populations is slow as compared to, and it is decoupled from, their aggregated dynamics. We employ a Taylor expansion in the small parameter ε scaling the strategy differences. It turns out that the orders of frequency-dependence (dependence of the growth rates on the relative abundances) are coupled to the orders in ε. Away from the singular points of the strategy space, the expansion is dominated by the linear term, which corresponds to frequency-independent selection and results in directional evolution. Dominance of the second order terms at a singularity leads to linear frequency dependence, i.e., to a Lotka-Volterra type relative dynamics. In both cases, outcome of competition is fully constrained by the invasion fitness. Pattern of coexistence in the vicinity of the singularity can be characterized.

Key Words: dynamical systems, population dynamics, adaptive dynamics

AMS Classification: 92D25, 92D15