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On the Canonical Equation of Adaptive Dynamics

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ABSTRACT

Adaptive Dynamics is an effective mathematical framework for dealing with long term biological evolution in "realistic" ecological settings. It differs from more classical approaches to modelling evolutionary change that assume constant fitnesses, by its focus on the population dynamical basis for those fitnesses, and hence on their inevitable change over evolutionary time. Of course the greater realism at the ecological end is bought by making a different set of (this time genetically unrealistic) simplifying assumptions, the two main ones being: (i) a separation of the population dynamical and mutational time scales, and (ii) clonal inheritance. One of the more powerful tools of AD is the so-called Canonical Equation, a differential equation describing how the trait (vector) dominating the population, changes over evolutionary time. The derivation of the CE is based on still one more simplifying assumption: (iii) small mutational step size. This CE has been derived in [1] for the from a biological viewpoint overly simplified case of ODE population models. In this talk I will present heuristic derivations for the case of general physiologically structures populations as treated in [2] (details of the derivation may be found in [3]) as well as for Mendelian populations. As it happens, the simple case turns out to be fairly representative!

Key Words: Adaptive Dynamics, Evolutionary Biology, Individual-based justification.

AMS Classification: 92D25, 92D15

References

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