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Uniqueness for optimal control problems with state equation arising from population dynamics

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ABSTRACT

In the paper [4], a survey about the last twenty years research concerning positive solutions for systems of nonlinear PDE's of Lotka-Volterra type is described. Also, there are considered some associated optimal control problems. While the uniqueness for optimal problems with one state equation of logistic type has, under suitable conditions, positive answer (see [3]), the case with a state system of Lotka-Volterra type is either more difficult or not considered ([1], [2], [4]). In this work, we will give conditions to assure the uniqueness of an optimal control problem studied in [4], where we maximize the cost-benefit functional $J : [0, \delta_1] \times [0, \delta_2] \subset L^\infty(\Omega) \times L^\infty(\Omega) \to \mathbb{R}$, defined as

$$J(f_1, f_2) = \int_\Omega \{K_1 u\, f_1 + K_2 v\, f_2 - M_1 f_1^2 - M_2 f_2^2\} dx,$$

where Ω is a bounded and regular domain in $\mathbb{R}^N$, K_1, K_2 are positive constants describing the prices of the prey and predator species, u, v respectively, and, M_1, M_2 are positive constants describing the cost of the controls f_1, f_2. The prey and predator population concentration, u, v, are modelled by the positive solutions of the corresponding Lotka-Volterra elliptic state system.

Key Words: Optimal control, Lotka-Volterra, nonlinear elliptic system, existence, uniqueness

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