

## Rich dynamics in a Lotka-Volterra predator-prey system with two delays

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### ABSTRACT

We consider the following Lotka-Volterra predator-prey system with two delays:

$$\begin{cases} x'(t) = x(t) [r_1 - ax(t - \tau_1) - by(t)] \\ y'(t) = y(t) [-r_2 + cx(t) - dy(t - \tau_2)]. \end{cases} \quad (\text{E})$$

The system dynamics becomes more and more complicated as time delay increases. First it is shown that a positive equilibrium of system (E) is globally asymptotically stable for small delays. Second, critical values of time delay through which system (E) undergoes a Hopf bifurcation are analytically determined. Some numerical simulations suggest the existence of subcritical Hopf bifurcation near the critical values of time delay. Finally system (E) exhibits some chaotic behavior as  $\tau_2$  increases when the degree of density dependence is relatively small compared to that of predations.

**Key Words:** predator-prey, subcritical Hopf bifurcation, chaotic behavior

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