International Workshop on Differential Equations in Mathematical Biology

Continuous Model for the Rock - Scissors - Paper Game between Bacteriocin Producing Bacteria

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ABSTRACT

In this work several aspects of bacteriocin producing bacteria and their interplay (see Riley and Wertz) are elucidated. Various attempts to model the Resistant, Producer and Sensitive E Coli strains in the so called RSP - Game (Rock - Scissors - Paper) have been made and the question arose, if there is a continuous model, which admits a cyclic structure like in fed batch cultures. The observations in experiments showed a cyclic structure in spatial distribution of these three competing species, while in mice cultures migration seemed to be essential for the reinfection in the RSP-cycle. This paper tries to describe the possible models and to rule out others, to clarify the underlying dynamics. In Szabó et. al. statistical effects (migration/mutation) seemed to be the driving force of the cycle, also in Kirkup and Riley spatial effects enable periodicity. In the Chemostat no periodic behavior was observed. On the other side fed batch systems can be constructed, which admit periodicity. The well known May-Leonard systems admit a Hopf bifurcation but it is degenerate and hence inadequate. Also the traditional RSP - game has no real counterpart. A parameter set was obtained for which a stable limit-cycle exists in the same Zeeman - class admitting a full cycle in the three-species game. These parameters are in accordance with the observed relations of the E Coli strains. Further studies will be made to analyze the stability of this cycle to get the representativity of the available space for which periodicity is possible. A complete parameter - space for a robust coexistence within this model is given.

Key Words: Lotka - Volterra - Systems, Hopf - Bifurcation, Bacteriocin, Rock - Scissors - Paper - Game

AMS Classification: 91A06, 91A12, 91A80, 37C27

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