

Polytopic Lyapunov functions for the stability analysis of persistence of competing species

F. Grogard¹, F. Mazenc², A. Rapaport²

¹INRIA Sophia-Antipolis, COMORE Project-team
2004 route des Lucioles, 06902 Sophia-Antipolis Cedex, FRANCE
frederic.grogard@inria.fr

²UMR Analyse des systèmes et biométrie
INRA-INRIA MERE Project-team
2, place Viala, 34060 Montpellier, FRANCE
{mazenc,rapaport}@ensam.inra.fr

ABSTRACT

In this paper, we will analyze the stability of the non-trivial equilibrium that arises in some mixed culture in competition for a single substrate. It is well known that, when the growth rate of the different species is Monod-like (strictly increasing from zero and upper-bounded), the generic equilibrium state for a given dilution rate consists in the survival of only one of the species (competitive exclusion [1]). This observation has been validated through laboratory experiments and in real life. This coexistence has been explained in different cases by a time -varying nutrient feed, multi-resource models, turbidity operating conditions or a crowding effect. It has been first shown in [2] that the coexistence of the different species can simply be explained by some form of intra-specific dependency of the growth function, which represents an intra-specific competition. We will here present a Lyapunov strategy for the proof of stability of that competition model. It will be based on the construction of a non-smooth Lyapunov function that explicitly takes into account the fact that the competition is linked to the sum of the concentrations of the competing species. It will then be generalized to more general kinds of competition.

Key Words: Chemostat, persistence, Lyapunov, asymptotically autonomous systems.

AMS Classification: 92A15, 92A17, 34C15, 34C35

References

[1] H.L. Smith and P. Waltman, *The Theory of the Chemostat*, Chapman and Hall, New-York, 1995.

- [2] C. Lobry, F. Mazenc, and A. Rapaport, Persistence in ecological models of competition for a single resource, *C. R. Acad. Sci. Paris, Ser I.*, vol. 340, 2005, pp. 199-204.