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Feedback stabilization for a chemostat with delayed output

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ABSTRACT

We study the feedback control of a simple model of chemostat described by the system:

$$\begin{cases} \dot{s} = D(s_{in} - s) - xf(s), \\ \dot{x} = x\{f(s) - D\}. \end{cases}$$

In this model, $s(t)$ denotes the concentration of nutrient and $x(t)$ denotes the density of the biomass at time t, f is a unimodal function and represents the per–capita growth of the biomass, $D > 0$ and $s_{in} > 0$ denote, respectively, the dilution rate of the chemostat and the concentration of nutrient.

We will suppose that D is the control variable and the nutrient is the only output available (*i.e* $y(t) = s(t)$). Our goal is to build a feedback control law $D(y(t))$ that stabilizes asymptotically the output at a reference value $s^* < s_{in}$. Applications to wastewater tratement and phytoplankton culture will be shown.

Considering that there exists a delay $\tau > 0$ in the measure of the nutrient, which implies $y(t) = s(t - \tau)$, the control of chemostat model (with D replaced by $D(t)$) becomes a system of differential delay equations. Therefore, our stabilization goal is reformulated as the asymptotic stability of s^* in an infinite dimensional setting. This control law increases the speed of convergence avoiding the oscillations that could be induced by the delay. Using asymptotically autonomous dynamical system theory, combined with some functional inequalities, we build a discrete dynamical system that inherits the asymptotic behavior of the original model, and we are able to find sufficient conditions for the global stability of the control system.

Key Words: Chemostat, Control theory, Differential delay equations

AMS Classification: 34K20, 37N35

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