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Monotonicity in Biological Systems: positive and negative feedback

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ABSTRACT

Many biological systems can be modelled by dynamical systems with fixed feedback relations among the variables: an increase in one component always leads to an increase (decrease) in another components rate of change. So-called monotone systems (competitive and cooperative systems) make up only a small fraction of these dynamical systems and for these one can make strong assertions concerning their asymptotic behavior. Motivated by the work of Sontag and coworkers in control theory, we show that certain finite and infinite dimensional, non-monotone, dynamical systems with negative feedback can be decomposed into a monotone "open loop" system with "inputs" and a decreasing "output" function. The original system is reconstituted by "plugging the output into the input". Employing a technique of Cosner of imbedding systems with fixed feedback relations into a larger symmetric monotone system, we are able to obtain information on the asymptotic behavior of solutions, including existence of positively invariant sets and global convergence. These results will be illustrated by applications to biological models.

Key Words: Monotone systems, monotone control systems

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