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TRANSIENT OSCILLATIONS INDUCED BY DELAYED GROWTH RESPONSE IN THE CHEMOSTAT

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ABSTRACT

In order to try to account for the transient oscillations observed in chemostat experiments, we consider a model of single species growth in a chemostat that involves delayed growth response. The time delay models the lag involved in the nutrient conversion process. Both monotone response functions and nonmonotone response functions are considered. The nonmonotone response function models the inhibitory effects of growth response of certain nutrients when concentrations are too high. By applying local and global Hopf bifurcation theorems, it is shown that the model has unstable periodic solutions that bifurcate from unstable nonnegative equilibria as the parameter measuring the delay passes through certain critical values and that these local periodic solutions can persist, even if the delay parameter moves far from the critical (local) bifurcation values.

Although these solutions are not themselves biologically meaningful, provided the initial data starts close enough to the unstable manifold of one of these periodic solutions they may still help to account for the transient oscillations that have been frequently observed in chemostat experiments. Numerical simulations are provided to illustrate that the model has varying degrees of transient oscillatory behaviour that can be controlled by the choice of the initial data.

Key Words: global asymptotic stability, nonmonotone response functions, delay, chemostat, inhibition.

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