ERRATA & NOTES

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Theory and Applications of Abstract Semilinear Cauchy Problems
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- p.21 In Assumption 1.1.27 $(A + \partial_x F(\mu, 0))_0$ should be replaced by $(A + \partial_x F(\mu, 0))$.
- p.22 In the Hopf bifurcation Theorem 1.1.28 $\varepsilon \to x_\varepsilon$ from $(0, \varepsilon^*)$ into $\mathbb{R}^n$ should be renamed $\varepsilon \to x_{0\varepsilon}$. Then the initial value $x_\varepsilon(0) = x_0$ should be replaced by $x_\varepsilon(0) = x_{0\varepsilon}$ (twice in the text and in the equation).
- p.92 l-8 $C_b([0, 1), \mathbb{R})$ should be replaced by $BC([0, 1), \mathbb{R})$.
- p.95 In Example 2.6.7. the notation $UBC(\mathbb{R}, \mathbb{R})$ should be replaced by $BUC(\mathbb{R}, \mathbb{R})$.
- p.113 l+4 "scalar product" should be replaced by "duality product".
- p.119 l+8 $drdl$ should be $dldr$
- p.119 l+9 $drdl$ should be dr
- p.119 l+11 The equation should be

$$(S_A * f)(t) = \int_0^t S_A(r) f(0) dr + \int_0^t \int_0^{t-r} S_A(r) f'(l) dl dr.$$

- p.135 In the Proposition 3.7.1. the last formula should be

$$\|\varphi\|_{L^p(J,Z)} = \sup_{\substack{\psi \in C_c^\infty(J,Z^*) \\ \|\psi\|_{L^q(J,Z^*)} \leq 1}} \int_J \psi(s) (\varphi(s)) \, ds.$$

- p.153 l+1 "Corollary 2.2.13" should be replaced by "Corollary 2.2.15"
- p.158 l+6 $(\lambda I - \mathcal{A})^{-1})$ should be replaced by $(\lambda I - \mathcal{A})^{-1}$
- p.164 *Kellermann and Hiber* should be replaced by *Kellermann and Hieber*

- p.189 There is a confusion between the index k used for the space E_k and T^k used in the part (b) of the proof of Theorem 4.3.16. There must be two different indexes. The proof reads as follows.

  Proof. (b) We prove $\dim(E_{k_0}) < +\infty$ by induction. Clearly $E_0 = \{0\}$. Thus,
$$\dim(E_0) = 0.$$
  Assume that $\dim(E_k) < +\infty$. Let $u \in B_{E_{k+1}}(0,1)$, then from part (a) of the proof we know that there exists $v \in E_k$ such that
$$Tu = u - v.$$
  We have
$$\|v\| \leq (1 + \|T\|) =: \delta$$
  and
$$T^2(u) = T(u) - T(v) = u - v - T(v)$$
  and by induction we obtain for each integer $m \geq 1$
$$T^m(u) = u - \sum_{l=0}^{m-1} T^l(v) \Leftrightarrow u = T^m(u) + \sum_{l=0}^{m-1} T^l(v).$$
  Hence
$$\kappa\left(B_{E_{k+1}}(0,1)\right) \leq \kappa\left(T^m(B_X(0,1)) + B_{E_k}(0,\delta) + TB_{E_k}(0,\delta) + ... + T^{m-1}B_{E_k}(0,\delta)\right)$$
  and, since $\dim(E_k) < +\infty$, we obtain
$$\kappa\left(B_{E_{k+1}}(0,1)\right) \leq \kappa\left(T^m(B_X(0,1))\right), \ \forall m \geq 1.$$
  When m goes to $+\infty$, since $r_{\text{ess}}(T) < 1$, it follows that $\kappa(T^m(B_X(0,1))) \to 0$. Thus,
$$\kappa\left(B_{E_{k+1}}(0,1)\right) = 0.$$
  It implies that $\overline{B_{E_{k+1}}(0,1)}$ is compact. But $(I-T)^{k+1}$ is bounded, we deduce that $E_{k+1} = \mathcal{N}((I-T)^{k+1})$ is closed, so is $B_{E_{k+1}}(0,1)$. Hence, $B_{E_{k+1}}(0,1)$ is compact. Now by applying the Riesz's theorem we obtain that $\dim(E_{k+1}) < +\infty$. □

- p.189 l-6 $X_n = \mathcal{R}((I-T)^n X)$ should be replaced by $X_n = \mathcal{R}((I-T)^n)$
- p.189 l-4 $f \in \mathcal{R}((I-T)X_k)$ should be replace by $f \in \mathcal{R}((I-T)|_{X_k})$.
- p.204 l-8 In Lemma 4.5.1. we mean $\forall \lambda \in \rho(A_Y)$.
- p. 222 l-11 In the proof of Lemma 5.2.3 (Uniqueness) $\delta(t)$ should be replaced by $\delta(t-t_0)$. Therefore the estimation should be
$$\|u(t) - v(t)\| \leq \delta(t-t_0) K(\tau + s, \xi) \sup_{l \in [t_0, t_0+t]} \|u(l) - v(l)\|.$$

- p.222-223 The statement of Lemma 5.2.4 and its proof is not correct. We should drop some $\delta(.)$ which was not there in the original result (see Lemma 5.4. in [1]). The original result and its proof should be the following.

  Lemma 0.1 (Local Existence)**.** *Let Assumptions 5.1.1, 5.1.2, and 5.2.1 be satisfied. Then for each* $\tau > 0$*, each* $\beta > 0$*, and each* $\xi > 0$*, there exists* $\gamma(\tau, \beta, \xi) \in (0, \tau_0]$ *such that for each* $s \in [0, \tau]$ *and each* $x \in X_0$ *with* $|x| \leq \xi$*, equation (5.1.1) has a unique integrated solution* $U(., s)x \in C([s, s + \gamma(\tau, \beta, \xi)], X_0)$ *which satisfies*

  $$|U(t,s)x| \leq (1+\beta)\xi, \ \forall t \in [s, s+\gamma(\tau,\beta,\xi)].$$

  Proof. Let $s \in [0, \tau]$ and $x \in X_0$ with $\|x\| \leq \xi$ be fixed. Let $\gamma(\tau, \beta, \xi) \in (0, \tau_0]$ such that

  $$\delta(\gamma(\tau,\beta,\xi))\, M\left[\widehat{\xi}_{\tau+\tau_0} + (1+\beta)\,\xi K(\tau+\tau_0, (1+\beta)\,\xi)\right] \leq \beta\xi$$

  with $\widehat{\xi}_\alpha = \sup_{s\in[0,\alpha]} \|F(s,0)\|, \forall \alpha \geq 0$. Set

  $E = \{u \in C([s, s+\gamma(\tau,\beta,\xi)], X_0) : |u(t)| \leq (1+\beta)\xi, \forall t \in [s, s+\gamma(\tau,\beta,\xi)]\}.$

  Consider the map $\Phi_{x,s} : C([s, s+\gamma(\tau,\beta,\xi)], X_0) \to C([s, s+\gamma(\tau,\beta,\xi)], X_0)$ defined for each $t \in [s, s+\gamma(\tau, p, C)]$ by

  $$\Phi_{x,s}(u)(t) = T_{A_0}(t-s)x + \frac{d}{dt}(S_A * F(.+s, u(.+s)))(t-s).$$

  We have $\forall u \in E$ that (using (5.2.1) repeatedly)

  $$\begin{aligned} |\Phi_{x,s}(u)(t)| &\leq \xi + M\left\|\frac{d}{dt}(S_A * F(.+s, u(.+s)))(t-s)\right\| \\ &\leq \xi + M\delta(\gamma(\tau,\beta,\xi)) \sup_{t\in[s,s+\gamma(\tau,\beta,\xi)]} \|F(t, u(t))\| \\ &\leq \xi + M\delta(\gamma(\tau,\beta,\xi))\left[\widehat{\xi}_\alpha + K(\tau+\tau_0, (1+\beta)\,\xi) \sup_{t\in[s,s+\gamma(\tau,\beta,\xi)]} |u(t)|\right] \\ &\leq (1+\beta)\,\xi. \end{aligned}$$

  Hence, $\Phi_{x,s}(E) \subset E$. Moreover, for all $u, v \in E$, we have (again using (5.2.1))

  $$\begin{aligned} &|\Phi_{x,s}(u)(t) - \Phi_{x,s}(v)(t)| \\ &\quad \leq M\delta(\gamma(\tau,\beta,\xi))\, K(\tau+\tau_0, (1+\beta)\,\xi) \sup_{t\in[s,s+\gamma(\tau,\beta,\xi)]} |u(t)-v(t)| \\ &\quad \leq \frac{K(\tau+\tau_0,(1+\beta)\,\xi)\beta\xi}{1+\widehat{\xi}_\alpha + K(\tau+\tau_0,(1+\beta)\,\xi)\,(1+\beta)\,\xi} \sup_{t\in[s,s+\gamma(\tau,\beta,\xi)]} |u(t)-v(t)| \\ &\quad \leq \frac{\beta}{1+\beta} \sup_{t\in[s,s+\gamma(\tau,\beta,\xi)]} |u(t)-v(t)|. \end{aligned}$$

[1] P. Magal, and S. Ruan (2007), On Integrated Semigroups and Age Structured Models in L^p Spaces, *Differential and Integral Equations* **20,2**, 197-139.

Therefore, $\Phi_{x,s}$ is a $\left(\frac{\beta}{1+\beta}\right)$-contraction on E and the result follows. $\square$

- p.229 The last inequality of Corollary 5.3.4. is only true for $x \in X_{0+}$. So it should be

$$\|U(t,s)x\| \leq e^{\gamma(t-s)} \left[C_1 \|x\| + C_2\right], \forall x \in X_{0+}.$$

- p.308 l-5 and l-6 the formula

$$\psi\left(\widehat{\theta}\right) = e^{\lambda\widehat{\theta}} (\lambda I - B)^{-1} \left[\alpha + \varphi(0)\right] - \int_0^{\widehat{\theta}} e^{\lambda(\widehat{\theta}-l)} \varphi(l)\, dl,\ \forall \widehat{\theta} \in [-r,0].$$

should be replaced by

$$\psi\left(\widehat{\theta}\right) = e^{\lambda\widehat{\theta}} (\lambda I - B)^{-1} \left[\alpha + \varphi(0)\right] - \int_{\widehat{\theta}}^{0} e^{\lambda(\widehat{\theta}-l)} \varphi(l)\, dl,\ \forall \widehat{\theta} \in [-r,0].$$

- p.309 l-11 $M_A := \sup_{t\geq 0} \left\|e^{(B-\omega_A I)t}\right\|_{M_n(\mathbb{R})}$ should be replaced by $M_A := \sup_{t\geq 0} \left\|e^{(B-\omega_A I)t}\right\|_{\mathcal{L}(\mathbb{R}^n)}$.