Second errata/addenda to the paperback edition 2006

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Page 13, Corollary 2.20: add "(i) implies that f is injective."

Page 22, in the commutative diagram, the horizontal arrows $\to 0$ are useless.

Page 32, Exercise 1.4(a): "Show that **every element of** $\mathfrak{p}A_{\mathfrak{p}}$ is nilpotent". In general $\mathfrak{p}A_{\mathfrak{p}}$ is only a nil ideal and not a nilpotent ideal.

Page 33, Exercise 1.8(a): "Any **point in the** preimage of a closed point is a closed point".

Page 39, Exercise 2.3(c): the presheaf $\mathcal{F}'$ satisfies Condition (4) in Definition 2.2.2, but not Condition (5) in general. So it is not a sheaf in general. The correct statement is $(\mathcal{F}')'$ is a sheaf.

Page 48, proof of Lemma 3.23: complete the proof as follows. Let V be an any open subset of Y. Let $a \in \mathcal{O}_Y(V)$. Then for any $x \in f^{-1}(V)$, $f^{\#}(V)(a)_x = f^{\#}_x(a_{f(x)}) = f^{\#}_{\varphi}(V)(a)_x$. It follows from Lemma 2.9 that $f^{\#}(V)(a) = f^{\#}_{\varphi}(V)(a)$ and $f^{\#} = f^{\#}_{\varphi}$.

Page 52, Proposition 3.38: add the fact that the stalk $\mathcal{O}_{\mathrm{Proj}B,\mathfrak{p}}$ at a homogeneous prime ideal $\mathfrak{p} \in \mathrm{ProjB}$ is isomorphic to the homogeneous localization $B_{(\mathfrak{p})}$.

Page 53, Lemma 3.40: replace $f|_{D_+(h)}$ by $f|_{D_+(\varphi(h))}$.

Page 67, Exercise 4.6(c): the notation $V(1-e)$ means complementary in X of X_{1-e} (see Definition 2.3.11). This notation is only defined in after 2.5.25.

Page 75, Exercise 5.4: (b)-(c): a quicker method is to pick for all $i \le r$ an element $x_i \in I \setminus \cup_{j \ne i} \mathfrak{p}_j$. Consider $x_1 + x_2 \cdots x_r$ and conclude.

Page 75, Exercise 5.5: we must suppose that I is generated by homogeneous elements of positive degree (or that none of the $\mathfrak{p}_i$'s contains A_+).

Page 76, Exercise 5.10: Prove the statement $X^0 \ne \cup_n Y_n^0$ only for **closed points** (otherwise the assumption on the base field k is useless).

Page 76, Exercise 5.11: add an example of an affine scheme of dimension 0 homeomorphic to $\{0,1\}^I$ (endowed with the product topology).

Page 77, Exercise 5.13: add the statement "any subextension of a function field is a function field, and the algebraic closure of k in a function field is finite over k".

Page 77, add a new exercise 5.16: an example of a quasi-projective curve X such that $f.\mathcal{O}_X(X) \neq (f\mathcal{O}_X)(X)$ for some $f \in \mathcal{O}_X(X)$.

Page 86, Exercise 1.5: add $\dim X_K = \dim X$ for any field extension K/k.

Page 86, Exercise 1.9: with the notation of (a), $k(u) \otimes_k k(v)$ is just the localization $T^{-1}k(v)[T]$. So it is a PID.

Page 88, proof of Proposition 2.2: we re-organize and add details. Here is the improved version.

Page 89, proof of Proposition 2.7(c): the proof can be simplified a little. Note that in the original proof, the equality $p(V(I)) = V(J)$ holds only after we prove the surjectivity of p.

Page 91, Corollary 2.14(c): in view of these discussions on Mathoverflow, it appears that we should clarify the notion of linear disjunction of extensions. This is done in the footnote here.

Page 96, Exercise 2.3: we **change the definition of immersions** and follow that of [EGA], I.4.1.3. The condition in our former definition is more restrictive, and is not stable by composition.

Page 107, Corollary 3.26: To be precise, $X(\mathcal{O}_K) = \mathrm{Mor}_Y(\operatorname{Spec} \mathcal{O}_K, X)$ and $X(K) = \mathrm{Mor}_Y(\operatorname{Spec} K, X)$. The scheme Z in the proof is considered as an $\mathcal{O}_K$-scheme.

Page 117, Example 1.19: in the last lines, replace m by $-m$ in $g_0^2 + mt_1 g_1^2$ and $G_0^2 + mt_1 G_1^2$

Page 117, Lemma 1.11: there is a much more direct proof: fix $\alpha \in I$ non-zero, then B is a submodule of $\alpha^{-1}I$ which is finitely generated over A. So B is finite over A.

Page 120, Proposition 1.22: "unique up to **unique** isomorphism".

Page 120-121, Definition 1.24: the word "extension" means a homomorphism of fields, not necessarily an inclusion. The normalization of X in L is unique up to **unique** isomorphism.

Page 121, 6th line in the proof of Proposition 1.25: "because L/K is **separable** (see [55], **VIII**, Theorem 5.2)".

Page 132, in the middle of the proof of Proposition 2.24: replace the portion between "$Z = \overline{\{x\}}$ of X" and "$y \notin V(J)$ (Jacobian criterion)." by the following: By Lemma 2.21, Z contains a regular closed (hence rational) point y. Let $\mathfrak{p}$ be the prime ideal of $A := \mathcal{O}_{X,y}$ defining x. Then $A_{\mathfrak{p}} = \mathcal{O}_{X,x}$ and $A/\mathfrak{p} = \mathcal{O}_{Z,y}$ are regular, thus Lemma 2.22 implies that $\mathcal{O}_{X,y}$ is regular.

Page 133, Exercise 2.1: for the second sentence ("x discrete in X_y"), suppose x is a closed point in X_y.

Page 136, Lemma 3.7: the conclusion is true without the finiteness hypothesis on the number of irreducible components of X. The proof goes in the same way by replacing B with the local ring of X at any generic point of X.

Page 138, end of proof of Corollary 3.14: X_y is equidimensional because $\dim \mathcal{O}_{X_y,x} = \dim X - \dim Y$ for all closed points x of X_y (let x run through the points belonging to only one irreducible component of X_y).

Page 139, Definition 3.17: the definition of unramified morphism is ambiguous concerning the extension $k(x)/k(y)$ of residue fields. Its finiteness is part of the defintion.

Page 143, proof of Proposition 3.38: add some details for the stability by base change.

Page 144, line 4: replace "Chapter 5" by "Chapter 6".

Page 145, Exercise 3.11: the statement is completely wrong as pointed out by Matt Emerton. Replace with:

- Let $f_1 : X_1 \to Y$, $f_2 : X_2 \to Y$ be morphisms of locally Noetherian schemes of finite type. Let us suppose that f_1 is unramified (resp. étale; resp. smooth) and surjective. Show that $X_1 \times_Y X_2 \to Y$ is unramified (resp. étale; resp. smooth) if and only if so is f_2.

Page 148, top line: "reduced (resp. **integral**)".

Page 152, proof of Corollary 4.8: in the last two lines of the proof, $\overline{X}_{\bar{f}} \to \operatorname{Spec} \mathcal{O}(\overline{X})$ is open and the latter is finite over Y. Then take $Z = \operatorname{Spec} \mathcal{O}(\overline{X})$ and $U = \overline{X}_{\bar{f}} \cap X$.

Page 155: Add some details at the end of the proof of Proposition 4.16.

Page 166, Lemma 1.22: The proof is rewritten.

Page 167, Theorem 1.27: $\mathcal{F}(n)$ is generated by **finitely many** global sections. This makes shorter the proof of 1.28.

Page 169-171:

1. Proposition 1.31: We put some more details in the proof of (a) and (b) and we add a new item (c) on very ample sheaves.

2. Definition 1.33, suppose X is either quasi-compact **and separated** or Noetherian.

3. The first part of the proof of Theorem 1.34 is incomplete if A is not Noetherian because the sheaf of ideals $\mathcal{J}$ is then not necessarily finitely generated. This part is now moved to Lemma 1.35(a). The proof of 1.35(b) used without mentioning Exercise 1.32. In the new version, it is non necessary. A statement (a') is added to 1.35.

4. Proposition 1.37: it seems that the proof is confusing. It is partly rewritten.

Here is the new version for 1.31, 1.34, 1.35 and 1.37.

Page 176, Exercise 1.24: add the assumption that $H^0(Y, \mathcal{O}_Y(1)) \to H^0(X, \mathcal{L})$ is surjective.

Page 178, Exercise 1.32: add the following statement “If X is Noetherian and has an ample sheaf, then X is separated”.

Page 194, Exercise 2.26(b): the result is true without assuming f finite. Use the (new version of) Lemma 1.35(a). Add Y separated.

Page 201, Corollary 3.16: add “X integral”.

Page 203, proof of the bijectivity of ρ_M: one can use the more basic Lemma 2.26 rather than Proposition 2.32 (two occurrences).

Page 205, Corollary 3.24: in the second part of the proof, replace (two occurrences) $H^1(X, (\mathcal{F} \otimes \mathcal{L}^{\otimes n})|_{X_{V_i}})$ by $H^1(X_{V_i}, (\mathcal{F} \otimes \mathcal{L}^{\otimes n})|_{X_{V_i}})$.

Page 205, Remark 3.25: It is also true for proper morphisms $X \to S$.

Page 214, Example 1.12: in the exact sequence, write $(C/P'(t)C)dT$ instead of $C/(P'(t))dT$ which could be ambiguous.

Page 218, Exercise 1.1: B is an A-**algebra**.

Page 219, Exercise 1.3: suppose from the beginning that A is noetherian and B essentially of finite type (i.e. localization of an algebra of finite type) over A. Actually, in (a), we need $\hat{B} = B + I\hat{B}$ which is not true in general if B is not noetherian.

Page 223, end of Proposition 2.10: "(resp. $y = f(x)$)". In the end of the proof, refer to Proposition 2.5 rather than Corollary 2.6 (which requires X, Y to be smooth in a neighborhood of x) and then use Exercise 4.3.8. Finally, $h^*\Omega^1_{Z/S} \to \Omega^1_{Y/S}$ is actually $h^*\Omega^1_{Y/S} \to \Omega^1_{Z/S}$.

Page 234, Proof of Corollary 3.24 and Exercise 3.4: we must suppose that A is catenary (Definition 8.2.1) in Exercise 3.4 except for the computation of $\dim A/(a_1, \ldots, a_r)$ in (a). In the proof of Cor. 3.24, Part (c) of this exercise is applied to the local ring $\mathcal{O}_{Z_y,x}$ which is catenary by Lemma 8.2.3(c) (or Proposition 2.5.23(a)).

Page 241, line 5: $(M/N) \otimes_C B$ instead of M/N.

Page 255, line 6: “multiplicative subset” instead of “(multiplicative) group”.

Page 262, in the last two lines of the proof of 1.33: permute $\mathcal{O}_X(U)$ and $\mathcal{O}_Y(V)$.

Page 266, after Exercise 1.13, add a new Exercise with an example where $\mathcal{K}_{X,x} \to \mathrm{Frac}(\mathcal{O}_{\mathrm{X,x}})$ is not an isomorphism (compare with Lemma 1.12(c)).

Page 270, Example 2.15: the rational function f has a zero at the point $q = (t = s = 1)$ of X. So we have to replace X by $X \setminus \{q\}$ (or replace D by the Cartier divisor $(X \setminus \{q\}, f)$, $(X \setminus \{p\}, 1)$).

Pages 271-272: Theorem 2.18 and Remark 2.19: In the left-hand side of the formulas, one has to replace the cycle by its codimension 1 part. The reason is that in general, f_* of a cycle of codimension 1 might not be of codimension 1 (Nagata), except when Y is universally catenary, 8.2.1, 8.2.6.

Page 278, proof of Proposition 3.13(a): replace "Exercise 5.2.16" by "Exercises 4.1.16 or 5.2.16(b)".

Page 280, Proposition 3.25(a): add $\dim_k H^0(X, \mathcal{O}_X) = l(0) \leq l(D)$. Note that $\dim_k H^0(X, \mathcal{O}_X) = 1$ if and only if X is, geometrically, reduced and connected.

Page 285, Proposition 4.1(a): "X is **isomorphic to** a smooth conic".

Page 286, line 7: $(s_i)_p \in p_i e + \mathfrak{m}_p e$.

Page 287, Lemma 4.6: replace $D \simeq i^*\mathcal{O}_P(m)$ with $\mathcal{O}_X(D) \simeq i^*\mathcal{O}_P(m)$.

Page 292, line -6: $z = s^r y$ and not $t^r y$.

Page 293, top line: $d = 2r - 1$ and not $2r + 1$.

Page 298, line 2: $G_T \to T$ instead of $G_T \to \operatorname{Spec} T$.

Page 304, Definition 5.3: we relax conditions on birational morphisms and adopt the general definition of [EGA], IV.6.15.4. This is to be coherent with Exercise 3.2.6 (where the morphism if not necessarily of finite type). But in this book we will only encounter birational morphism of finite type between Noetherian, and mostly integral, schemes.

Page 306, end of the proof of Proposition 5.7: "As $M_{\mathfrak{p}_i} = A_{\mathfrak{p}_i}/\mathfrak{p}_i A_{\mathfrak{p}_i}$".

Page 309, bottom line: change $\Delta(k^{*m})$ to $\Delta(k^*)$.

Page 322-323, Propositions 1.12 and 1.15: replace "quasi-coherent sheaf of ideals" by "coherent sheaf of ideals". Of course as we work on locally noetherian schemes, this does change nothing. This is just for consistency with Definition 1.11.

Page 332, after Exercise 1.8, add a new exercise (the blowing-up is birational).

Page 333, Remark 2.2: add the equivalence: for any irreducible closed subsets $T \subseteq Z$, any maximal chain of irreducible closed subsets between T and Z has lenght $\mathrm{codim}(T, Z)$. Note also that a closed subscheme of a catenary scheme is catenary.

Page 335, Example 2.10: $\operatorname{depth} \mathfrak{p} = 0$ if and only if $\mathfrak{p}$ is contained in (not equal to) some $\mathfrak{q} \in \mathrm{Ass}(A)$.

Page 336, Lemma 2.12(c): it is a redundant with 2.12(b) and 2.13.

Page 337, Proposition 2.15(c): it comes immediately from 2.13.

Pages 351-352, Lemma 3.9: in (a), X_s as Weil divisor is not defined ! See the new text including, just above Lemma 3.9, the definition of the cycle associated to a closed subschme. The statement of 3.9 is rewritten.

Page 353, last line of the proof of Theorem 3.16: we need a version of Corollary 5.3.24 for proper (instead of projective) morphism. See errata above for Remark 5.3.25, page 205.

Page 354, in the line next to the commutative diagram: replace Theorem 3.3.25 by Corollary 3.3.26.

Page 355, Definition 3.21: “normal fibered surface**s**”.

Page 355, Proposition 3.22: “if ξ is **a** generic point” (the strict transform of x needs not be irreducible). The connectedness of the strict transform $f(x) = p_2(p_1^{-1}(x))$ is a consequence of Corollary 5.3.16 which says that $p_1^{-1}(x)$ is connected.

Page 357-359: Add an assertion in Proposition 3.28. The proofs of Lemma 3.29 and Proposition 3.30 were ugly and incomplete in some places. They are fairly better now. Change Remark 3.31. See the new pages.

Page 361, Theorem 3.42: replace “a reduced scheme that is locally of finite type over an excellent, reduced, locally Noetherian, scheme ...” (which is exactly “a reduced excellent scheme”) by “a reduced Noetherian excellent scheme ...”.

Page 366, middle of the page: replace $y_2 = t_2^{e-2}x^e u$ by $y_2 = t_2^e x^{e-2} u$.

Page 373, Figure 13: it probably misses a component of multiplicity 5 between $6\Gamma_6$ and $4\Gamma_4$. I haven’t not yet checked.

Page 380, Proposition 1.11: the regularity hypothesis is not necessary and the proof is even simpler. This makes Exercise 9.1.1 useless.

Page 381, lines -3 and -2: replace “bilinear” by “linear”.

Page 395, proof of Lemma 2.1: to show that we have a morphism $X \to \tilde{Y}$, by the universal property of the blowing-up, it is enough to show that $m_y\mathcal{O}_X$ is invertible at points $x \in X_y$. But $\mathfrak{m}_y\mathcal{O}_{X,x}$ defines a closed subscheme of codimension 1, so it has height 1. As $\mathcal{O}_{X,x}$ is a UFD, $\mathfrak{m}_y\mathcal{O}_{X,x}$ is principal, hence invertible.

Page 397-398, Lemma 2.10 and Proposition 2.11: the statement and the proof of 2.10(b) is improved, and 2.11 is corrected as in Theorem 7.2.18. See the new text. Proposition 2.11 is used in Theorem 2.12 where Y is regular, hence universally catenary (Corollary 8.2.16).

Page 401, Lemma 2.17: add “(c) if T is regular, then $\mathrm{Pic}(T) \to \mathrm{Pic}(U)$ is an isomorphism”. In the proof of Proposition 2.18, we use the surjectivity of $\mathrm{Pic}(X) \to \mathrm{Pic}(X \setminus \{x\})$.

Page 406, Example 2.33: $y^2z + x^3 + tz^3$ instead of $yz^2 + x^3 + tz^3$.

Page 409, Exercise 2.8(a): recall that r_Γ is defined in Exercise 9.1.9.

Page 413, first line of the proof of 3.4: Let $\mathcal{L}$ be a **very** ample sheaf...

Page 418, Proposition 3.13: replace $\mathrm{Aut}_{X_\eta}(X_\eta)$ by $\mathrm{Aut}_{K(S)}(X_\eta)$.

Pages 421-422: Modify Lemma 3.20 (the former statement is correct but the proof is incomplete) and the proof of Theorem 3.21. Here is the new text.

Page 424: Change the proof of Proposition 3.32 using the new version of Lemma 3.20.

Page 429, line 2 of §9.4: "(Corollary **3**.26 and Exercise **3**.2)".

Page 444–445: Lemma 4.29: the statement is re-organized and the proof is partly rewritten. Here is the new text.

Page 469, Definition 1.39: "dropping the reference to $\mathcal{X}$ (**not** x) in the notation...".

Page 514, proof of Corollary 3.22(a): as S is not necessarily local, $\mathcal{O}_{K'}$ (which is finite étale over $\mathcal{O}_{S,s}$) may not be of finite type over S. But of course $\operatorname{Spec} \mathcal{O}_{K'}$ is a localization of Dedekind scheme of finite type over S.