(a) Show that one can suppose that $X = \operatorname{Spec} B$, $Y = \operatorname{Spec} A$, and V are affine.

(b) Let I, J be the ideals of B defining X_1 and X_2. Tensoring the exact sequence $0 \to A \to \mathcal{O}_Y(V)$ by B/I, show that $I = \operatorname{Ker}(B \to \mathcal{O}_X(f^{-1}(V))/(I))$. Deduce from this that $I = J$.

3.8. Let $f : X \to Y$ be a morphism of locally Noetherian schemes. Let Z be a closed subscheme of X. We suppose that there exists a point $y \in Y$ such that $Z_y = X_y$ as schemes.

(a) Show that if Z is flat over Y at $z \in X_y$, then Z equals to X in a neighborhood of z (use Exercise 1.2.14 and Nakayama's lemma).

(b) Show that if Z is flat over Y and if f is moreover of finite type (resp. proper) over Y, then there exists an open neighborhood $V \ni y$ such that $Z \cap f^{-1}(V) \to f^{-1}(V)$ is an open immersion (resp. an isomorphism) (use Exercise 3.2.5).

3.9. Let $f : X \to Y$ be a flat morphism of finite type with X, Y Noetherian. We will show that f is open.

(a) Use Exercise 3.2.17 to show that $f(X)$ contains a non-empty open subset V of Y.

(b) By considering the morphism $X \times_Y (Y \setminus V) \to Y \setminus V$, show that $f(X) \setminus V$ contains a non-empty open subset of $Y \setminus V$.

(c) Show that $Y \setminus f(X)$ is closed in Y by using the fact that the topological space Y is Noetherian; that is, every descending chain of closed subsets in Y is stationary.

3.10. Let k be a field of characteristic $\operatorname{char}(k) \neq 2$. Let $Y = \operatorname{Spec} k[u, v]/(v^2 - u^2(u+1))$, and let $f : X \to Y$ be the normalization morphism. Show that f is unramified, surjective, but not étale.

3.11. Let $f_1 : X_1 \to Y$, $f_2 : X_2 \to Y$ be morphisms of locally Noetherian schemes of finite type. Let us suppose that for every $y \in Y$, there exists $i = 1$ or 2 such that f_i is unramified (resp. étale; resp. smooth) at all points of $f_i^{-1}(y)$. Show that $X_1 \times_Y X_2 \to Y$ is unramified (resp. étale; resp. smooth).

3.12. Let $f : X \to Y$ be a flat morphism of algebraic variety. We suppose that X_y is irreducible for every closed point $y \in Y$ and that Y is irreducible. Show that X is irreducible.

3.13. Let X be a smooth morphism over a scheme S of characteristic $p > 0$.

(a) Show that $X^{(p)} \to S$ is smooth (use Proposition 3.38).

(b) Let us suppose that S is the spectrum of a field k. Let $F_{X/k} : X \to X^{(p)}$ be the relative Frobenius. Show that $F_{X/k}$ is flat at the rational points of X (use Proposition 2.27).

(c) Show that $F_{X/k}$ is *faithfully flat*, that is flat and surjective.