CM Jacobians

Abstract: We study, for a given value g, the Torelli locus T_g inside the moduli space A_g of polarized abelian varieties, and we like to understand which "canonical coordinates" on A_g can be induced on T_g. This question can be made more precise (as it turns out):

QUESTION. Which curves of genus g give a Jacobian $J = \mathrm{Jac}(C)$ that is a CM abelian variety?

CONJECTURE (Coleman, 1987). For $g > 3$ the number of CM curves over the complex numbers is finite.

We will see that this stimulating conjecture gave rise to the study of deeper structures. We will

– discuss examples (A. J. de Jong & R. Noot; G. Shimura; FO) showing that for $3 < g < 8$ the conjecture does not hold true;

– formulate an approach to this question via the André-Oort conjecture;

– compare results by Dwork-Ogus and Fresnel-Van der Put approaching these questions via deformation "from the boundary";

– show that the conjecture by Coleman holds true (under GRH) if we restrict to a slightly smaller class of Jacobians (joint work with Ching-Li Chai);

– formulate several questions, problems and an expectation. (In short: the talk will contain more open problems than solved ones.)

Material and references for this talk can be found:

[M-O] B. Moonen & F. Oort, *The Torelli locus and special subvarieties.* The Handbook of Moduli (G. Farkas, I. Morrison, editors), Vol. **II**, pp. 545–590. [To appear in 2012.]

[C-O] C.-L. Chai & F.Oort, *Abelian varieties isogenous to a Jacobian.* Ann. Math. **176** (2012), 589–635. [To appear in 2012.]

A detailed description of this talk will be posted on:
  `http://www.staff.science.uu.nl/~oort0109/`