

# Constraint Programming course : projects

Ruslan Sadykov

February 16, 2022

## Rules

- You can work in groups of maximum 2 people.
- You should write a report of  $\approx$  3–4 pages, in which you describe your model.
- At most one group can choose each project (the rule is « first come, first serve »).
- You should return the report on the eve of the exam day (TD surveillé) at latest (an electronic version by e-mail suffices). Attach the code to the report (or include it to the report).  
Please send to **ruslan.sadykov@inria.fr**.
- The last two TP sessions (3rd and 10th of March) will be devoted to projects. On these sessions you may ask all questions related to your project. The advice is to have a model for your project and an idea how to implement it by the end of the last TP session.

## Project 1. Eternity

*Eternity* is a puzzle competition ([https://en.wikipedia.org/wiki/Eternity\\_II\\_puzzle](https://en.wikipedia.org/wiki/Eternity_II_puzzle)). You have to place  $n \cdot m$  square pieces into a  $n \cdot m$  grid, constrained by the requirement to match adjacent edges. Each puzzle piece has its edges marked with a color. The border does not have any color and represented with the symbol « - ».

*Project aim.* Model this problem as a CSP and try to solve the following test instance using a solver of your choice. Test different heuristics for the variable order and present computational results of their comparison.

*Test instance.* Each line describes a piece using 4 letters representing the colors clockwise. - means that this edge should be adjacent to the border.

```
- G R -  
V V Y V  
A V V Y  
Y A V Y  
V Y Y V  
G - - G  
- B V G  
Y Y A V  
A V Y V  
A V V A  
B - G Y  
R V O -  
Y Y V Y  
V Y V Y  
V V Y Y  
Y G - R  
- O Y B  
Y Y A Y  
Y V A V  
O - B V  
O Y R -  
V Y V Y  
V Y V V  
Y R - B  
- O Y O  
Y V Y Y  
R - O V  
R Y B -  
V V Y Y  
V B - O  
- B Y O  
R - R V  
B Y G -  
Y O - G  
- - R B  
G G - -
```

*Remark :* It is important to determine a good heuristic for variables instantiation.

*Maximum mark.* 18.

## Project 2. Aircrew assignment

The problem consists in assignment of the aircrew (air hostesses and stewards) to flights of an air company.

- There are 20 employees. Stewards : *Tom, David, Jeremy, Ron, Joe, Bill, Fred, Bob, Mario, Ed*, air hostesses : *Carol, Janet, Tracy, Marilyn, Carolyn, Cathy, Inez, Jean, Heather, Juliet*.
- There are 10 flights.
- Number of employees assigned to each flight is fixed :

|                |   |   |   |   |   |   |   |   |   |    |
|----------------|---|---|---|---|---|---|---|---|---|----|
| Flight number  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Aircrew number | 4 | 5 | 5 | 6 | 7 | 4 | 5 | 6 | 6 | 7  |

- Aircrew of each flight should include at least a certain number of air hostesses and stewards :

|                              |   |   |   |   |   |   |   |   |   |    |
|------------------------------|---|---|---|---|---|---|---|---|---|----|
| Flight number                | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Min. number of are hostesses | 1 | 1 | 1 | 2 | 3 | 1 | 1 | 1 | 2 | 3  |
| Min. number of stewards      | 1 | 1 | 1 | 2 | 3 | 1 | 1 | 1 | 2 | 3  |

- Aircrew of each flight should include at least one person who speaks french, at least one person who speaks spanish, and at least one person who speaks german. Knowledge of languages :

Français    *Inez, Bill, Jean, Juliet*  
 Espagnol    *Tom, Jeremy, Mario, Cathy, Juliet*  
 Allemand    *Bill, Fred, Joe, Mario, Marilyn, Inez, Heather*

- If we assign somebody to a flight, we cannot assign him (or her) to two following flights.

*Project aim.* Model this problem as a CSP and solve it using a solver of your choice. Try to use more global constraints.

*Maximum mark : 15.*

### Project 3. Orchestra rehearsal

The concert consists of 9 music compositions. Each composition involves 5 members of the orchestra. Each music player can arrive immediately before the first composition in which he is involved and leave immediately after its last composition. One needs to find the order of compositions for a rehearsal such that the total waiting time of players is minimized.

The problem data is presented in the following table. The cell  $(i, j)$  contains  $\times$  if player  $i$  is involved in composition  $j$ .

In addition, during the rehearsal, composition 2 should be played before composition 8 and composition 6 should be played immediately after composition 5.

| Composition number   | 1        | 2        | 3        | 4        | 5        | 6        | 7        | 8        | 9        |
|----------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| Player 1             | $\times$ |          | $\times$ | $\times$ |          | $\times$ | $\times$ |          | $\times$ |
| Player 2             | $\times$ |          | $\times$ | $\times$ | $\times$ | $\times$ |          | $\times$ |          |
| Player 3             | $\times$ | $\times$ |          |          | $\times$ |          |          | $\times$ |          |
| Player 4             | $\times$ |          |          |          | $\times$ |          | $\times$ |          | $\times$ |
| Player 5             |          | $\times$ |          | $\times$ | $\times$ | $\times$ | $\times$ |          |          |
| Composition duration | 2        | 4        | 1        | 3        | 3        | 2        | 5        | 7        | 6        |

If the order is  $(1, 2, 3, 4, 5, 6, 7, 8, 9)$ , the waiting time for player 1 is 14  $(4 + 3 + 7)$ , for player 2 — 9  $(4 + 5)$ , for player 3 — 11  $(1 + 3 + 2 + 5)$ , for player 4 — 17  $(4 + 1 + 3 + 2 + 7)$  and for player 5 — 1. The total waiting time is then equal to 52.

*Project aim.* Model this problem as a CSP and solve it using a solver of your choice. Finding a good heuristic for variables instantiation is important. Detect the symmetries in your model and eliminate them.

*Maximum mark :* 18.

### Project 4. Stable marriage problem

Each man in a group of 6 is asked to classify each of 6 women using a preference order. Every man necessarily classifies all women. In the same way, every woman classifies using preference order all men.

We call *mariage* a set of 6 couples such that every man marries exactly one woman and every woman is married to a exactly one man. A marriage is *instable* if it contains two couples  $(m, w)$  and  $(m', w')$  such that  $m$  prefers  $w'$  to  $w$  and  $w'$  prefers  $m$  to  $m'$ , otherwise the marriage is *stable*.

The preference orders for men is the following (smaller number indicates greater preference) :

| Men     | Women |       |       |       |       |      |
|---------|-------|-------|-------|-------|-------|------|
|         | Helen | Tracy | Linda | Sally | Wanda | Mary |
| Richard | 3     | 5     | 4     | 2     | 1     | 6    |
| James   | 3     | 2     | 5     | 6     | 4     | 1    |
| John    | 2     | 4     | 3     | 1     | 5     | 6    |
| Bill    | 5     | 6     | 4     | 2     | 3     | 1    |
| Greg    | 2     | 5     | 3     | 6     | 4     | 1    |
| Mario   | 1     | 3     | 4     | 5     | 6     | 2    |

The preference orders for women :

| Women | Men     |       |      |      |      |       |
|-------|---------|-------|------|------|------|-------|
|       | Richard | James | John | Bill | Greg | Mario |
| Helen | 2       | 4     | 5    | 3    | 6    | 1     |
| Tracy | 3       | 5     | 4    | 2    | 1    | 6     |
| Linda | 1       | 3     | 6    | 2    | 4    | 5     |
| Sally | 3       | 2     | 5    | 6    | 4    | 1     |
| Wanda | 6       | 4     | 2    | 1    | 3    | 5     |
| Mary  | 6       | 4     | 3    | 1    | 5    | 2     |

The problem is to find a stable marriage.

*Project aim.* Model this problem as a CSP and solve it using a solver of your choice. In addition, please verify whether there exists a stable marriage such that each woman is married to at most 4th man in her preference order.

*Maximum mark : 16.*

## Project 5. Repair shop scheduling

Our repair shop has received 12 orders today. We have 3 employees, each on can repair only one device at a time. Fixing jobs cannot be interrupted. To start to repair device  $j$ , on needs  $r_j$  minutes to obtain spare parts for it. The fixing job duration for device  $j$  is  $p_j$  minutes. Afterwards, each device should be delivered to the client. Deliveries are performed by another company and can be done in parallel. Delivery of device  $j$  takes  $l_j$  minutes. We can postpone fixing jobs to the next day. The postponement of device  $j$  raises the penalty of  $w_j^1$  euros. One have also a possibility to speed up by 20% the fixing job for every device  $j$ . Such speed-up costs  $w_j^2$  euros.

All operations (delivery, fixing jobs) should be done during working hours (8 hours = 480 minutes). The objective is to schedule the fixing jobs while minimising the total postponement penalty.

The problem data :

| Orders : | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $p_j$    | 140 | 80  | 160 | 120 | 160 | 120 | 130 | 100 | 40  | 140 | 160 | 60  |
| $r_j$    | 20  | 30  | 40  | 50  | 100 | 10  | 20  | 40  | 100 | 10  | 50  | 20  |
| $l_j$    | 120 | 150 | 170 | 100 | 140 | 100 | 150 | 230 | 100 | 90  | 180 | 280 |
| $w_j^1$  | 100 | 80  | 120 | 100 | 80  | 120 | 120 | 90  | 100 | 80  | 150 | 130 |
| $w_j^2$  | 30  | 20  | 40  | 20  | 10  | 30  | 40  | 10  | 40  | 20  | 30  | 50  |

*Project aim.* Model this problem as a CSP and solve it using a solver of your choice. If possible, detect the symmetries in your model and eliminate them.

*Maximum mark :* 17.

## Project 6. Engine plant scheduling

We run a plant which produces engines. We have a package of orders for different engines. Each engine consists of components. A component may also consist on other components. For producing an engine/component, all components of which it consists should be available. Producing an engine/component consumes some resources during a certain time. Our factory is working 24/24 hours. However, it closes for the week-end (from 5 :00PM on Friday to 9 :00AM on Monday). An operation cannot be interrupted for the week-end break.

So, the data for the problem is the following. For each order we know the ordered engine type and the due date. For each resource we know its capacity (i.e., the total number of available units of this ressource). For each engine or component we know the list of required (preceding) components and the list of required resources. Every component needs one unit of every required ressource during a specified time. When production of a component starts, all required ressources should be available. The ressources are freed after a specified time, which can be different for different required ressources (even for the same component).

The problem consists in scheduling operations for producing components. The objective is to minimize to total tardiness for all order. The tardiness of an order is the delay of engine production (non-negative difference between the due date and engine production date).

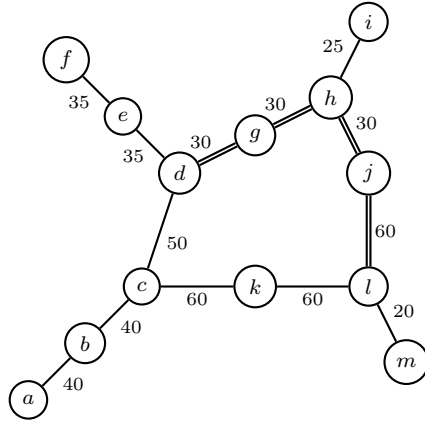
The data will be sent directly to groups which choose this project.

*Project aim.* Model this problem as a CSP and solve it using a solver of your choice.

*Maximum mark :* 18.

## Project 7. Train scheduling

The dispatcher should schedule the trains for a part of the railways. This part is modeled by a graph  $G = (V, E)$  where set  $V$  of vertices represents the railway stations and set  $E$  of edges represents the railway lines which trains can take. The length of an edge equals to the time (in minutes) needed for passing through the corresponding railway line.



Connections between stations consist of a single railway line, except certain connections marked with the double line. Therefore, if two trains take the same single railway line in different directions, one after another, the second train cannot start before the first one finishes this line. If two trains take the same railway in the same direction (including double connection), for security reasons, the second train can start at least 10 minutes after the departure of the first one.

For each train  $t$  we know its starting station  $i_t \in V$  and its arrival station  $j_t \in V$ . For each train we also know the release time  $r_t$  (the time when the train is ready to depart), and due time  $d_t$  (desired time when the train should reach its destination). Each train should absolutely follow the fastest path (the shortest path on the graph).

| Trains | $i_t$ | $j_t$ | $r_t$ | $d_t$ |
|--------|-------|-------|-------|-------|
| 1      | $f$   | $a$   | 0h00m | 4h00m |
| 2      | $i$   | $a$   | 1h00m | 4h30m |
| 3      | $i$   | $m$   | 0h30m | 3h30m |
| 4      | $m$   | $a$   | 1h00m | 5h00m |
| 5      | $a$   | $j$   | 3h00m | 6h00m |
| 6      | $a$   | $f$   | 2h00m | 5h30m |
| 7      | $c$   | $m$   | 1h30m | 4h00m |
| 8      | $h$   | $f$   | 0h30m | 3h30m |
| 9      | $m$   | $g$   | 1h00m | 5h00m |
| 10     | $m$   | $d$   | 1h30m | 5h00m |
| 11     | $f$   | $i$   | 2h30m | 5h00m |



The problem is to schedule the trains, i.e. for every train, one need to define its starting time for each railway line (edge) it takes. The objective is to minimise that total tardiness, i.e. the sum, for each train, of the difference between its arrival time and its due time.

*Project aim.* Model this problem as a CSP and solve it using a solver of your choice.

*Maximum mark : 20.*

## Project 8. Traveling Tournament Problem

The Traveling Tournament Problem with Predefined Venues (TTPPV) consists of finding an optimal compact single round robin schedule for a sport championship. Given a set of  $n$  teams, each team has to play against every other team exactly once. In each round, a team plays either at home or away, however no team can play more than three consecutive times at home or away. The sum of the traveling distance of each team has to be minimized. The particularity of this problem resides on the venue of each game that is predefined, i.e. if team  $a$  plays against  $b$  it is already known whether the game is going to be held at team  $a$  home or at team  $b$  home.

For this project, we consider the symmetric circular distances, i.e. the distance between homes of teams  $a$  and  $b$ ,  $a \leq b$ , (given the total number of teams equal to  $n$ ) is

$$d_{ab} = d_{ba} = \min\{b - a, a - b + n\}.$$

The data will be sent directly to groups which choose this project.

*Project aim.* Model this problem as a CSP and solve it using a solver of your choice.

*Maximum mark : 17.*

### **Project 9. Your own problem**

You can propose your own problem. It should be validated by the teacher.  
The maximum mark depends on the problem proposed.