Constraint Programming
Lab 4.

27 January 2022

1 Representing integers

What domains are represented by

1. $\{[[x \leq 6]], \neg[[x \leq 2]]\}$
2. $\{[[x \leq 9]], \neg[[x \leq 4]], \neg[[x = 6]], \neg[[x = 8]]\}$
3. $\{[[x = 4]]\}$
4. $\{[[x \leq 5, \neg[[x \leq 4]]\}$
5. $\{[[x \leq 7, \neg[[x \leq 1]], \neg[[x = 8]]\}$
6. $\{[[x = 4]], [[x = 7]]\}$
7. $\{\neg[[x \leq 7]], [[x \leq 1]]\}$

2 Explanations

Give the resulting domain and explanation for each of the following examples

1. $D(x_1) = \{2, \ldots, 4\}$, $D(x_2) = \{1, \ldots, 4\} : x_1 + 1 \leq x_2$
2. $D(x_1) = D(x_2) = D(x_3) = D(x_4) = \{1, 2\} : \texttt{all-different}(x_1, x_2, x_3, x_4)$
3. $D(x_1) = \{2, 3\}$, $D(x_2) = \{1, 4\}$, $D(b) = \{false, true\} : b \Leftrightarrow x_1 = x_2$
4. $D(x_1) = D(x_2) = \{1, \ldots, 4\}$, $D(x_3) = \{3\}$, $D(x_4) = \{1, \ldots, 4\} : 2x_1 + x_2 + 3x_3 + x_4 \leq 12$

3 Lazy Clause Generation

— $D(x_1) = D(x_2) = D(x_3) = D(x_4) = D(x_5) = D(x_6) = \{1, \ldots, 5\}$, $D(b_1) = D(b_2) = \{false, true\}$
— Constraints
  — $b_1 \vee b_2$
  — $b_1 \Leftrightarrow x_1 \geq x_6$
  — $b_2 \Leftrightarrow x_1 \geq 4$
  — $x_1 + x_2 + x_3 + x_4 \leq 11$
  — $x_4 \geq x_5$
  — $x_3 \geq x_5$
  — $x_5 + x_6 \leq 8$
— Assume decisions in order : $x_6 \geq 4$, $x_5 \geq 2$, $x_2 \geq 4$
— Build the implication graph, determine the 1UIP (First Unique Implication Point) Nogood (which is a clause). Show the result after backjumping