Constraint Programming
Exercise Lab 5.

3 February 2022

1 Golomb ruler with symmetry

A **Golomb ruler** is a set of integers in which the distances between elements are pairwise different. For a given integer n, a Golomb ruler is « optimum » if its length (maximum distance) is minimum. Examples of optimum Golomb rulers :

$n = 2$	0 1
$n = 3$	0 1 3
$n = 4$	0 1 4 6
$n = 5$	0 2 7 8 11

Model the problem to find an optimum Golomb ruler for a given integer n as a CSP. Try to use global constraints. Determine and remove symmetries of your model, if possible.

2 Latin square with symmetry

A **Latin square** is an $n \times n$ array filled with n different symbols, each occurring exactly once in each row and exactly once in each column..

An example of a 4×4 Latin square is :

$$
\begin{bmatrix}
1 & 3 & 2 & 4 \\
4 & 2 & 1 & 3 \\
2 & 4 & 3 & 1 \\
3 & 1 & 4 & 2
\end{bmatrix}
$$

1. Model the problem of finding an $n \times n$ Latin square.
2. Find and eliminate the symmetries of your model, if exist.

3 Balanced Incomplete Block Designs

In this problem, we are given five numbers as an input (v, b, r, k, l). The goal is to construct a $0 - 1$ matrix with v rows and b columns such that there are exactly r ones per row, k ones per column, and the scalar product of every two rows is l.

An example of solution for $(v, b, r, k, l) = (7, 7, 3, 3, 1)$ is

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Model this problem. Find and eliminate the symmetries of your model.

4 Scene allocation

You are a movie producer and you need to shoot a film which involves a certain number of actors. For each actor, we know the set of scenes he plays in. At most k scenes can be shot per day. Each actor is paid by the day : if he plays in at least one scene shot this day, he is paid for this day. The pay for each actor is different.

The goal is to assign each scene of the film to a certain day and to minimize the total pay to actors. Model this problem. Find and eliminate the symmetries of your model.