Quasi-linear electron acceleration in a driven plasma wave

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Abstract

Saturation of the electron plasma wave amplitude exited by a spatially localized driver and the electron acceleration are studied by means of numerical simulations. The model consists of the Zakharov equations for the electron plasma and ion acoustic waves coupled to the quasi-linear equation for the electron distribution function. The saturation levels of both electron plasma and ion acoustic waves and the number and energy of hot electrons are studied in function of the driver wave number and amplitude and the size of the excitation region. A correlation between the onset of the strong Langmuir turbulence and the efficient electron heating is discussed.

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I. INTRODUCTION

Knowing the saturation level of electron plasma wave (EPW) is an important issue in many plasma applications. In particular, in the context of inertial confinement fusion by lasers, the wave-particle interactions can accelerate electrons to high energies that may cause a preheat of the fusion fuel and reduce the target gain [2]. The EPWs can be generated in the resonance absorption of the laser light near the critical density or by various parametric instabilities such as the stimulated Raman scattering (SRS), the two-plasmon decay, and the ion acoustic decay. The EPWs interact between themselves and the ion acoustic waves (IAWs) via the Langmuir decay instability (LDI), which is the interaction of a primary and secondary EPWs with a resonant IAW [3]. The EPWs can grow to large amplitudes and often bring plasma into a turbulent state. They also interact efficiently with electrons and may accelerate them to high energies. Recent experimental results concerning the LDI process in the laser driven plasmas can be found in Ref. [4].

Numerical simulations of the SRS-driven Langmuir turbulence for a long time where based on the fluid equations and the generalized Zakharov models [5, 6]. The Zakharov equations [7] can describe the wave-wave processes that result in a cascade of the EPW energy to low wave numbers as well as the modulational instability and nucleation processes that result in a formation of coherent wave packets and their subsequent collapse in self-generated density depletions. In spite of a considerable success, it is commonly understood that the Zakharov model suffers from deficiency related to the hypothesis of a fixed EPW damping. Therefore the electron heating was not accounted for in such models. Vlasov [8] or particle-in-cell (PIC) [9, 10] simulations have been used for a more complete description of the problem. However, these kinetic simulations have difficulties in studying weak instabilities and a long time behavior, because they are forced to resolve very small (electron) spatial and temporal scales.

A quasi-linear-Zakharov model was introduced by Thomson *et al.* [11] and further explored by Dubois *et al.* [12] in order to account for the wave-wave and wave-particle processes simultaneously without using numerically demanding PIC simulations. This is a hybrid model, which includes the kinetic effects by coupling the Zakharov equations to the electron quasilinear diffusion equation. Although this model has a limited domain on applicability due to the random phase approximation for the EPW spectrum and the linear treatment of the IAW response, it is simple and flexible enough to investigate the processes in large time and spatial scales and to accommodate additional physical effects (for example, the multiple dimensions, inhomogeneity, etc.). It has much lower numerical noise compared to the PIC codes and it is much faster then the kinetic Vlasov codes.

In this paper, we use the quasilinear-Zakharov model with an external, spatially-localized monochromatic driver for studies of the nonlinear saturation of the EPW turbulence and the electron acceleration in a broadband spectrum of EPWs. The organization of this paper is as follows. The model equations used in the simulations are presented in Section II. The high frequency Langmuir field is described by the generalized Zakharov equation with the Landau damping dependent on the tail of electron distribution function. The latter is given by the electron quasi-linear diffusion equation for the electron distribution function averaged of the interaction domain. The self-consistent low frequency ion density response is described by a linear IAW equation with a fixed linear Landau damping and with the ponderomotive force from the high frequency Langmuir field. The energy to the system is supplied by a spatially-localized source term in the equation describing the EPWs. This localization of the driver in space allows us to study the effect of convection of plasma waves energy from the source. Some details of the numerical techniques for solving these equations are also discussed.

In Section III, we recall the effect of three-wave interactions on the wave evolution in Zakharov equations. Section IV is devoted to a numerical analysis of the EPW evolution and the electron acceleration in function of the driver wavelength, its amplitude, and the size of the localization region. For each regime, we consider the time history of the spatially averaged Langmuir wave energy and the electron energy and the evolution of the electron distribution function. We demonstrate that the wave-wave (LDI) and the wave-particle interactions occur simultaneously and they control the redistribution of the absorbed energy between the electrons and the waves. Section V presents our conclusions.

II. QUASI-LINEAR-ZAKHAROV MODEL

The quasi-linear-Zakharov (QLZ) model couples a quasi-linear diffusion equation for the spatially averaged electron distribution function to the modified Zakharov equations, evolving the EPW and IAW amplitudes. This model was derived by DuBois *et al.* [12] from the Vlasov-Poisson equations by using an oscillation center transformation that decomposes the distribution function into the envelope component which vary slowly on the plasma frequency time scale and a high-frequency response. In the following, we recall the principal points of the model derivation and discuss its conservation properties.

A. Modified Zakharov equations

The theoretical model is based on the system of coupled equations for the EPWs and IAWs. The Langmuir field is described in the envelope approximation, the amplitude E varies in time on the ion acoustic scale and is related to the high frequency electrostatic field $\mathcal{E}$ by $\mathcal{E}(x,t) = \mathrm{Re}\, E(x,t)\, \mathrm{e}^{-i\omega_{pe}t}$ where $\omega_{pe} = (n_0 e^2/\epsilon_0 m_e)^{1/2}$ is the electron plasma frequency. It depends of the electron distribution function via the Landau damping term and couples to the ion acoustic waves via density perturbations. The ion density is described by a linear IAW equation (without dispersion and with a fixed Landau damping) and it is coupled to the Langmuir field by the ponderomotive force. Assuming that the density is given in the form $n = n_0 + \delta n$, where n_0 is a constant background electron density and δn the fluctuation, the Zakharov wave equations read:

$$\partial_t E + \nu_e * E - i\frac{3}{2}\omega_{pe}\lambda_{De}^2 \partial_x^2 E = -\frac{i}{2}\omega_{pe}\frac{\delta n}{n_0}E - \frac{i}{2}k_p v_{the} S\, \exp(ik_p x - i\Omega_p t), \tag{1}$$

$$\partial_t^2 \delta n + 2\nu_i * \partial_t \delta n - c_s^2 \partial_x^2 \delta n = \frac{c_s^2 \epsilon_0}{4T_e}\partial_x^2 |E|^2, \tag{2}$$

where $v_{the} = \sqrt{T_e/m_e}$ and $c_s = \sqrt{ZT_e/m_i}$ are the electron thermal and ion acoustic velocities, respectively and they are assumed to be time independent; $\lambda_{De} = v_{the}/\omega_{pe}$ is the electron Debye length, k_p and $\Omega_p = \frac{3}{2}k_p^2\lambda_{De}^2\omega_{pe}$, are the wave number and the frequency of the primary Langmuir wave driven by the source S, which is a function defining the spatial localization of the driver.

On the left hand side of Eq. (1), the third term represents the thermal dispersion, the second term, $\nu_e * E$, describes the Landau damping or growth depending on the slope of the spatially averaged electron velocity distribution function $F_e(v,t)$. The damping term is known in the Fourier space

$$\hat{\nu}_e(k,t) = -\frac{\pi\omega_{pe}^3}{2n_0 k\,|k|}\,\partial_v F_e\left(\frac{\omega_{pe}}{k}, t\right).$$

Positive $\hat{\nu_e}$ represents damping and negative one represents amplification. When a Maxwellian electron velocity distribution function is assumed, $F_{e0}(v) = (n_0/v_{the})\ (2\pi)^{-1/2}\ \exp(-v^2/2v_{the}^2)$, the linear Landau damping takes the standard form:

$$\hat{\nu}_e(k) = \sqrt{\frac{\pi}{8}}\,\frac{\omega_{pe}}{|k\lambda_{De}|^3}\exp\left(-\frac{3}{2} - \frac{1}{2(k\lambda_{De})^2}\right).$$

The IAW equation (2) follows from the ion fluid equations in a linear approximation and assuming the quasi-neutrality ($\delta n = Z\delta n_i = \delta n_e$), that is, neglecting the dispersion effect. This equation describes the density fluctuations with the frequency $\omega_a(k) = |k|c_s$ induced by the ponderomotive force of the localized high frequency fields. The linear IAW damping $\hat{\nu}_i(k)$ depends on the ion distribution function. Both the electron and ion damping terms in (1) and (2) are convolution operators applied in the Fourier space. In the present model we account for electron distribution function modification, while the ion distribution function is fixed.

B. The electron quasi-linear diffusion equation

The Zakharov wave equations are completed by the electron quasi-linear diffusion equation for the spatially averaged velocity distribution function $F_e(v,t) = (1/L)\int_0^L dx\; f_e(x,v,t)$. It is obtained by taking the spatial average of the Vlasov equation over the length of the simulation domain L:

$$\partial_t F_e = \partial_v \left[D(v,t)\partial_v F_e\right] - \frac{|v|}{L}(F_e - F_{e0}), \qquad D(v,t) = \frac{e^2}{8|v|Lm_e^2}\left|\widehat{E}\left(\frac{\omega_{pe}}{v},t\right)\right|^2, \tag{3}$$

where D is the diffusion coefficient in the phase space. The Maxwellian electron distribution was considered as the initial condition. The last term in the right hand side of (3) was proposed by Sanbonmatsu *et al.* [15] in order to account for losses of fast electrons from the interaction region due to its finite width in the perpendicular plane. In the present formulation, we consider a spatially localized driver of the width less than ten wavelengths. The main energy losses are due to the wave and particle energy transfer off the excitation region. Moreover, we use spatially periodic boundary conditions in the diffusion equation (3) and the term accounting for the losses in the perpendicular direction will break down the particle conservation. For these reasons in the reference set of simulations, we set these losses to zero. In the last section, we will return to this assumption and discuss of the effect of two-dimensional loses.

Equation (3) describes the evolution of the particule distribution function in the range of velocities that correspond to the phase velocities of the EPWs. The quasi-linear diffusion has a tendency to flatten the electron distribution and to reduce the Landau damping. However, due to the flux of slower particles in the resonant region, the Landau damping eventually increases above the initial Maxwellian value. Because of strong Landau damping the diffusion coefficient $D(v,t)$ is zero for sufficiently small velocities, $v \lesssim (2-3)v_{the}$, therefore the quasilinear diffusion involves only the tail of electron distribution. This allows us to neglect the bulk electron heating and consider the electron thermal velocity in the Zakharov equations as constant.

The quasi-linear theory is valid if two conditions are satisfied. The first one is the phase incoherence, that is, the EPW spectrum supposed to be sufficiently wide and the phases of individual harmonics to be statistically uncorrelated. The second condition is the time scale ordering: the quasi-linear diffusion approximation is only valid if the particle diffusion time is much longer than the autocorrelation time of the wave spectrum [15]. The applicability conditions depend essentially on the driver wavenumber [14]. For the small enough $k_p \lambda_{De} < 0.2$, these conditions could be questioned at the initial stage evolution, when the driven EPW is quasi-monochromatic, however, in later times, after a few cascades of LDI the spectrum broadens and the quasi-linear approximation becomes completely valid. For $k_p \lambda_{De} > 0.3$, the strong Landau damping suppresses the LDI, the spectrum od EPWs remains narrow and the quasi-linear approximation could produce incorrect results [15]. Below we are considering the case of a spatially localized driver which assures the applicability of the quasi-linear approximation at the initial stage. Nevertheless we keep the driver wavenumber below $0.3\,\lambda_{De}^{-1}$ so the strong Landau damping does not suppress completely the LDI and allows the nonlinear evolution of the EPWs.

C. Conserved quantities

The full quasi-linear Zakharov system have the following conserved quantities :

1. The total number of electrons in the system, $d_t N_e(t) = 0$, where the total number of electrons $N_e(t)$ is defined as

$$N_e(t) = \iint dx dv\, f_e(x,v,t) = L \int dv\, F_e(v,t).$$

2. The total number of ions in the system, $d_t N_i(t) = 0$, where the total number of ions $N_e(t)$ is defined by the quasi-neutrality condition

$$N_i(t) = \int_0^L dx\, (n_0 + \delta n).$$

3. The total energy of the electron system changes due to EPW coupling to the driver.

$$d_t W_{tot}(t) = \frac{1}{2} k_p v_{the} \epsilon_0 \int dx\, \mathrm{Im}\, E^* S\, \mathrm{e}^{ik_p x - i\Omega_p t}, \tag{4}$$

where the energy $\mathcal{W}(t)$ is defined as a sum of the wave and particle energies

$$W_{tot} = \frac{1}{2}\epsilon_0 \int dx\, |E|^2 + \frac{1}{2} m_e \int dv\, v^2 F_e$$

and the right hand side of (4) accounts for the energy input from the driver.

4. Also the H-theorem may be demonstrated from equation (3) by multiplying it by $1 + \ln F_e$ an integrating over the velocity space. This gives

$$-d_t\, H_F = \int dv\, F_e^{-1} D\, |\partial_v F_e|^2 \geq 0,$$

where $H_F(t) = -\int dv\, F_e \ln F_e$ is the entropy.

These quantities are used as additional diagnostics in the numerical simulations.

D. Numerical algorithm

Many numerical methods have been proposed for the Zakharov part of the system. A spectral method was designed by Payne *et al.* [16]. They used a truncated Fourier expansion in their algorithm to eliminate aliasing errors. An energy-preserving finite difference scheme for the Zakharov system in one dimension was presented by Glassey [17]. We apply a time-splitting spectral approximation for the Zakharov equations and we restrict ourselves to periodic boundary conditions for both the EPW and IAW fluctuations. This allow us to use the Fourier spectral method. Let E_j^m and δn_j^m be the approximations of $E(x_j, t_m)$ and $\delta n(x_j, t_m)$, respectively. From time $t = t_m$ to t_{m+1}, one solves equation (1) for the time step, the length $\Delta t = t_{m+1} - t_m$ without the coupling term $\delta n\, E$. It is discretized in space by the Fourier spectral method and integrated in time exactly. In the second step of length Δt, we solve the rest of the equation (1) and the equation (2) with initial data given by the solution of the equation for E at the previous step.

We discretized (2) in space by the Fourier spectral method and integrated in time exactly. To get E^{m+1}, we approximated the integral of δn on $[t_m\, t_{m+1}]$ via the trapezoidal rule. The electron quasilinear diffusion equation is discretized with an implicit difference scheme by using an *ad-hoc* velocity mesh grid defined as $v_j = \omega_{pe}/k_j$ with $k_j = 2\pi j/L$.

III. LANGMUIR DECAY INSTABILITY AND LANGMUIR TURBULENCE

The primary EPW at with the wavenumber k_p and the frequency $\Omega_p = \frac{3}{2}\omega_{pe}(k_p\lambda_{De})^2$ is excited by the driver S at the interval of a length ΔL. The amplitude of this wave grows until it exceeds the LDI threshold. It is a resonant three-wave interaction that transfers the energy from unstable EPW into the backward propagating EPW at $k_1 = -k_p + \delta k$ and the IAW at $k_{ia} = 2k_p - \delta k$. The factor $\delta k \lambda_{De} = (2/3)\, c_s/v_{the}$ accounts for the energy transmitted to the IAW. It can be calculated from conditions of the frequency matching, $\omega_{EPW0} = \omega_{EPW1} + \omega_{IAW1}$ and the wave number matching, $k_{EPW0} = k_{EPW1} + k_{IAW1}$.

As the amplitude of the scattered EPW1 grows, it becomes sufficiently energetic to act as a pump for the second cascade of the LDI. This process continues for a number of back and forward cascades. At each step, the wave number is reduced by the amount δk. The result is that the wave energy flows to lower wave numbers. Since the phases of scattered waves are not correlated, after a few subsequent cascades one creates a weak turbulence spectrum that have been observed in numerical solutions to the Zakharov equations [5, 6]. If there is a sufficient dissipation at wave numbers lower than k_p, the primary instability can be saturated, and the resulting state is a weak EPW-IAW turbulence. If the dissipation in the domain of small wave numbers is insufficient,the energy is built up at long wavelengths. This provokes the modulational instability that assumes the energy transport toward smaller scales by means of spatially collapsing wave packets. The modulational instability and the EPW collapse are both manifestations of a strong Langmuir turbulence, also described by Zakharov equations.

The collapse causes an eventual transfer of energy up to large wave numbers, where the wave-particle interaction becomes important and Landau damping provides a sufficient dissipation that saturates the primary instability. This implies that the absorbed energy is transferred to electrons, changes their velocity distribution function and the Landau damping. With time going on, the perturbations of the distribution function spread towards larger velocities and modify completely the initial spectrum of the EPW turbulence. The purpose of this study is to obtain an asymptotic state of the electron distribution function, the EPW and IAW energy spectra and investigate how this energy repartition between these two subsystems depends on the wavelength and the amplitude of the localized driver.

IV. SIMULATION RESULTS

We present the results for several sets of parameters that sweep through regimes of interest. These parameters are the driver wave number k_p, the width of the excitation zone ΔL and the driver amplitude S. Other parameters such as the velocity ratio, $v_{the}/c_s = 45$ and the interaction box length, $L = 6000\ \lambda_{De}$, remain fixed. In particular, the temperature ratio, $ZT_e/T_i = 10$, was sufficiently large so the IAW damping was neglected. The initial EPW electric field and the amplitude of low frequency density perturbation in the Zakharov equations are set to zero. The initial electron distribution is assumed to be a Maxwellian function. The driver amplitude was time independent and has a Gaussian spatial shape, $S(x) = S_0\ \exp[-(x - L/4)^2/2\Delta L^2]$. We were using a 8192 grid points with with the Fourier mode spacing $\Delta k = 2\pi/L \approx 10^{-3}\ \lambda_{De}^{-1}$.

The system evolution will be characterized by following set of results :

- Dimensionless EPW energy: $W_E(t) = (\epsilon_0/2n_0T_eL)\ \int |E|^2 dx$.

- Dimensionless increase of the electron energy: $W_e(t) = (m_e/2n_0T_e)\ \int v^2 F_e(v,t) dv - 1/2$.

- Dimensionless energy density of EPWs: $U_E = \epsilon_0 |E|^2 / 2 n_0 T_e$.
- Spectrum of EPWs : $U_k = \epsilon_0 |E_k|^2 / 2 n_0 T_e L^2$, where $E_k = \int_0^L dx\, E(x,t)\, \mathrm{e}^{-ikx}$.
- Relative low-frequency perturbation : $N = \delta n / n_0$.
- Spectrum of density fluctuations : $N_k = |\delta n_k|^2 / n_0^2 L^2$, where $\delta n_k = \int_0^L dx\, \delta n(x,t)\, \mathrm{e}^{-ikx}$.

A. Reference set of parameters

The reference set of parameters corresponds to a regime of a weak drive with the dimensionless amplitude $\overline{S_0} = S_0/\sqrt{n_0 T_e/\epsilon_0} = 7.6 \times 10^{-3}$, the driver wavenumber $k_p \lambda_{De} = 0.12$, and the width of the excitation region $\Delta L = 250\, \lambda_{De}$.

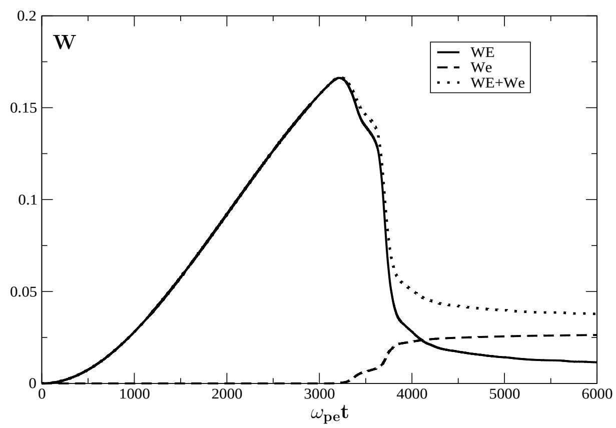

FIG. 1: Time history of the spatially averaged EPW energy W_E (solid line), the excess of the electron energy W_e (dashed line) and the total deposited energy $W_{tot} = W_E + W_e$ (dotted line) for the reference set of parameters.

Figure 1 shows the time history of the spatially averaged EPW energy, W_E, and the electron energy W_e. There are three characteristic stages in the evolution: (i) the linear stage where the external driver excites the primary EPW at k_p, (ii) the saturation stage where the EPW energy decreases strongly as the LDI cascade begins at $\omega_{pe} t = 3200$, and (iii) the asymptotic stage where the driver becomes decoupled and the absorbed energy is distributed between the turbulent EPWs and the electrons. At the second stage a significant part (about 70%) of the absorbed energy returns back to the driver and the wave energy saturates near $\omega_{pe} t = 5500$, at the much lower level due to the interplay between the wave-wave and wave-particle processes. The saturation of the electron energy in Fig. 1 occurs almost simultaneously with the wave energy saturation. It increases during the transient period and does not change at the asymptotic stage.

The particle and wave spectra give a more detailed information about the processes. The quasi-linear diffusion starts early in the linear stage (Fig. 2 dotted line) but it involves only a narrow zone near the phase velocity. The energy range $(30-45)\,T_e$ corresponds to the width of the frequency spectrum of the driven EPWs around the energy corresponding to the phase velocity, $\varepsilon_{ph} = \frac{1}{2}T_e/(k_p\lambda_{De})^2$. Then the diffusion domain broadens quickly at the end of the LDI stage of evolution. Figure 2 shows that the diffusion first spreads to higher energies as the LDI

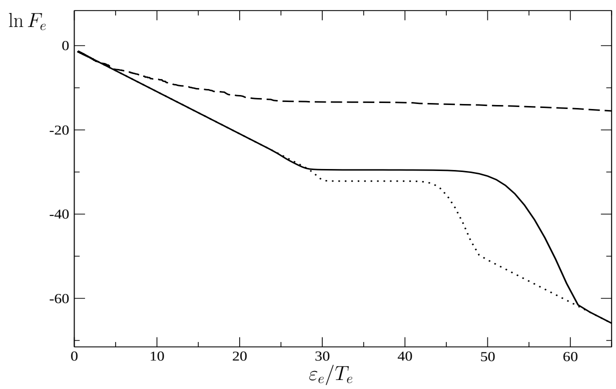

FIG. 2: Averaged electron distribution function at time $\omega_{pe}t = 1000$ (dotted line), 2000 (solid line), and 6000 (dashed line) for the reference set of parameters.

generates longer wavelengths EPWs. Than the diffusion moves toward the lower energies which is the signature of the modulation instability and the EPW collapse. The number of particles in the tail increases at the second stage of the quasi-linear diffusion at $\omega_{pe}t = 3200-3800$, and it attains the level of a few tenths of percent. The accelerated electrons have an approximately Maxwellian distribution with the temperature $T_h \simeq 9\,T_e$, which is significantly lower than the energy corresponding to the driver phase velocity.

The flattening of the electron distribution function at the initial stage of the EPW excitation has an important effect on the EPW growth. This process is called the quasi-linear inflation [12, 14], it decreases the Landau damping in the range of wave numbers close to k_p and enhances the mode growth which attains a nonlinear level in a shorter time scale. This can be seen in the EPW and IAW spectra shown in Figs. 3 and 4.

The EPW spectrum is narrow in first two panels. It is defined by the driver spectral width and it grows in time linearly. The correspondent spectrum of IAWs in Fig. 4 describes the density depletion in the excitation zone by the ponderomotive force. This density depletion creates a nonlinear shift of the driven EPW and saturates its amplitude at the time $\omega_{pe}t_{sat} \simeq 3000$. The saturation amplitude can be estimated directly from Eq. (1) as $E_{p\,sat}\,\delta n_{sat}/n_0 \simeq k_p\lambda_{De}S_0$ while

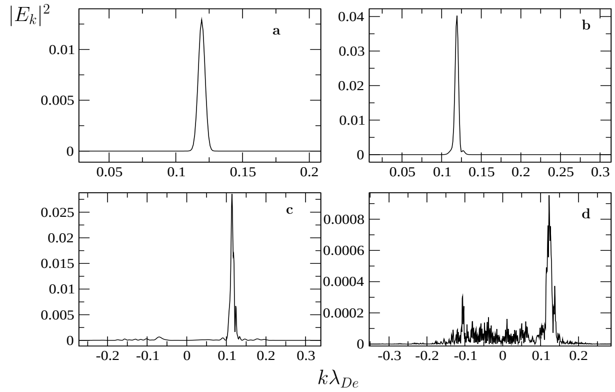

FIG. 3: The Langmuir wave spectra at time $\omega_{pe}t = 1000$ (a), 2000 (b), 3200 (c), and 5000 (d) for the reference set of parameters.

the value of the density depletion follows from Eq. (2): $\delta n_{sat} \sim \epsilon_0 E_{p\,sat}^2/4T_e$. Combining these two relations one can estimate the EPW saturation amplitude:

$$\frac{\epsilon_0 E_{p\,sat}^2}{4n_0 T_e} \simeq (k_p \lambda_{De})^{2/3} \left(\frac{\epsilon_0 E_p^2}{4n_0 T_e} \right)^{1/3} \tag{5}$$

and the saturation time, $\omega_{pe} t_{sat} \simeq 2\, n_0/\delta n_{sat}$. The first LDI begins almost at the same time and manifest itself in the supplementary peaks in panels c in Figs. 3 and 4. Then the following LDI cascade steps are excited (k_p, $-k_p + \delta k$, $k_p - 2\delta k, \ldots$) for the EPWs and the corresponding IAWs ($2k_p - \delta k, \ldots$). The characteristic time scale is given by the LDI growth rate $\gamma_{LDI} = (\omega_{pe} k_p c_s \epsilon_0 E_p^2/8n_0 T_e)^{1/2}$. It was verified that the simulations at the linear stage reproduce well the theoretical value of the growth rate. The value of γ_{LDI} defines the duration of the second stage of the EPW evolution which takes about $(3-4)\,\gamma_{LDI}^{-1}$.

The distribution of the wave energy in the real space is shown in Figs. 5 and 6. The EPW and the density perturbation grow initially in the domain where the driver is localized. Note that despite of a relatively small width of the excitation region, less than ten wavelengths, the convective effect is negligible and the saturation occurs due to the nonlinear frequency shift. The beginning of the LDI at $\omega_{pe}t = 2800$ is manifested in the real space it a small-scale structure on the EPW space profile. The LDI cascade increases the EPW intensity, consequently the ponderomotive force increases the density depletion a several times and the amplitude of the driven EPW wave falls down about ten times (Fig. 1). This scenario of the EPW saturation is the same as in the Zakharov model, the only difference is that the quasi-linear inflation accelerates the mode growth and decreases the saturation time.

Later on the EPW amplitude and density perturbation are spread out spatially and split in

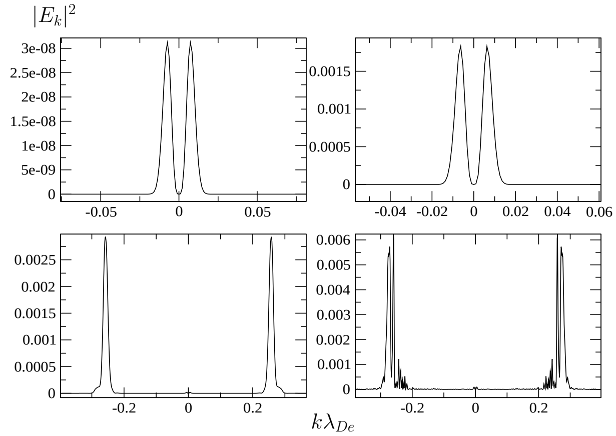

FIG. 4: Ion acoustic spectra at time $\omega_{pe}t = 50$ (a), 2000 (b), 3200 (c), and 5000 (d) for the reference set of parameters.

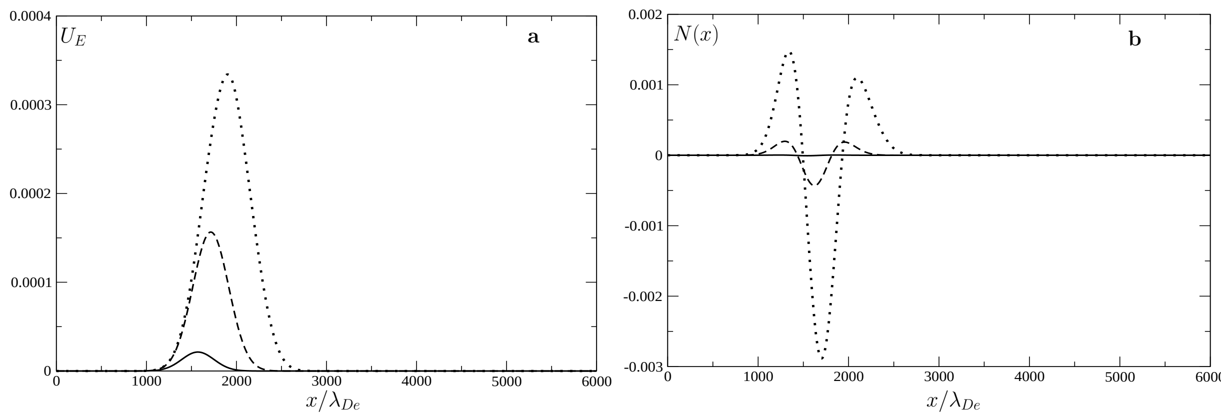

FIG. 5: Distribution of the EPW (a) and IAW (b) energy in the real space for the reference set of parameters: solid line $\omega_{pe}t = 200$, dashed line $\omega_{pe}t = 1000$, and dotted line $\omega_{pe}t = 2000$.

many localized intense regions. The EPW energy W_E further decreases due to the increased Landau damping as it is shown in Fig.7. Moreover, starting from the time $\omega_{pe}t = 3400$, the EPW energy propagates in both sides from the source region.

B. Dependence of the EPW saturation on the driver parameters

The scenario of the EPW saturation due to the combined effect of the LDI and the quasi-linear electron diffusion presented in the previous section is rather robust and exists for a relatively broad range of parameters. In this section we investigate the effects of variation of the driver wavenumber, while keeping the driver potential $\bar{S}_0 = 7.6 \times 10^{-3}$ and the width of the excitation

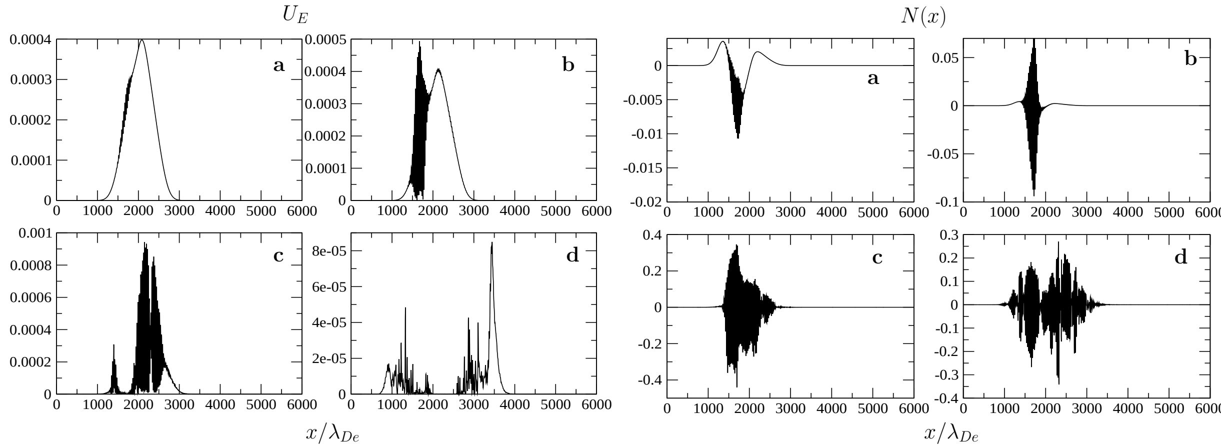

FIG. 6: Spatial distribution of the EPW (left) and IAW (right) energy for the reference set of parameters for $\omega_{pe}t = 2800$ (a), 3000 (b), 3200 (c) and 3400 (d).

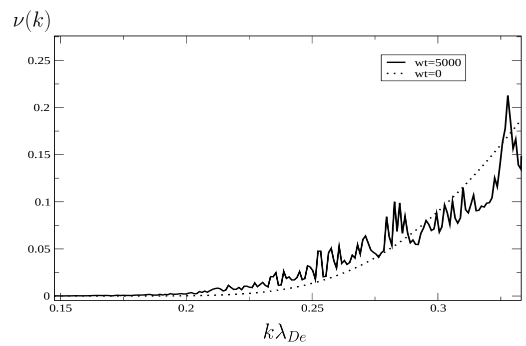

FIG. 7: EPW Landau damping rate in function of the wave number: dashed line $\omega_{pe}t = 0$, solid line $\omega_{pe}t = 5000$.

zone $\Delta L = 250\,\lambda_{De}$ unchanged. Essentially the same dependencies were found for higher driver amplitudes and for a wider excitation regions. In all cases a stronger drive produces a faster initial growth and a quicker saturation at a higher level.

Figure 8 shows the time history of the spatially averaged EPW energy W_E and the electron energy W_e for two values of $k_p\lambda_{De} = 0.15$ and 0.26. The latter value is close to the limit of applicability of the quasi-linear theory. According to [14], for larger k_p the spectral width of the driven EPW becomes so small that the quasi-linear approximation fails and the particle trapping in a quasi-monochromatic EPW takes over. The maximum level of W_E increases with $k_p\lambda_{De}$ and the hot electron temperature decreases. In the contrary, the level of the EPW and the electron energy asymptotic saturation demonstrate non-monotonic dependence on the driver wavelength.

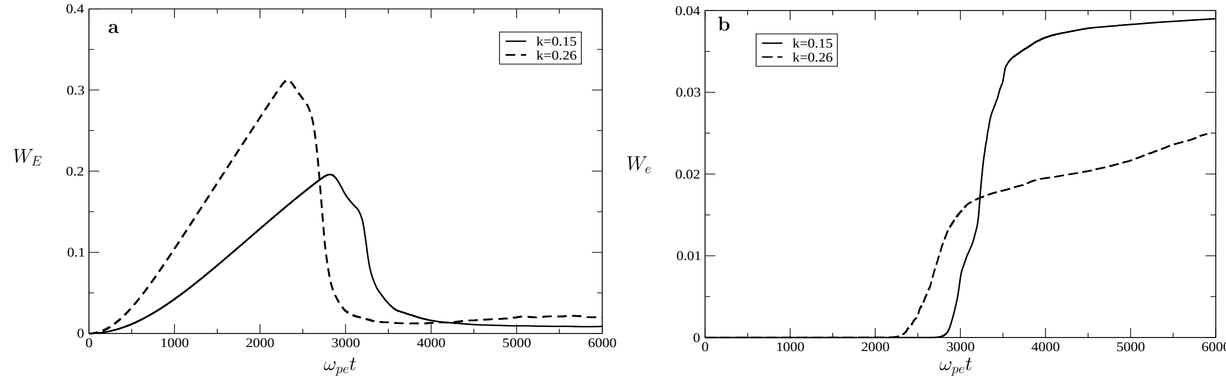

FIG. 8: Time evolution of the spatially averaged EPW energy (a) and the electron energy (b) for $k_p\lambda_{De} = 0.26$ (dashed line) and 0.15 (solid line). The driver width $\Delta L\lambda_{De} = 250$ and the amplitude $\bar{S}_0 = 7.6 \times 10^{-3}$.

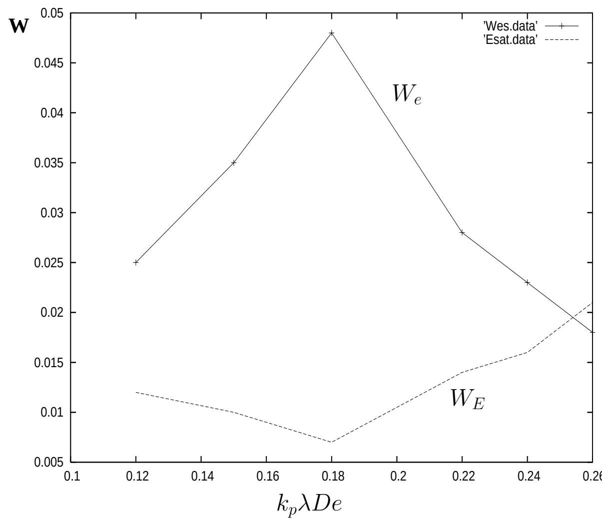

FIG. 9: Spatially averaged electron energy (solid line) and the EPW energy (dashed line) at the final time moment $\omega_{pe}t = 5000$ in function of the driver wave number $k_p\lambda_{De}$ for the driver amplitude $\bar{S}_0 = 7.6 \times 10^{-3}$ and $\Delta L\, \lambda_{De} = 250$.

That dependence is shown in more details in Fig. 9. The EPW energy attains its minimum and the electron energy its maximum for $k_*\lambda_{De} \simeq 0.18$. Such a dependence on k_p can be understood from the wave spectra and the evolution of the electron distribution function. From the EPW spectra shown in Fig. 10 one can see that the significant difference between $k_p > k_*$ and $k_p < k_*$ stems from the fact that in the former case the number of LDI cascades is limited and the spectrum is localized in short wavelength domain. The number of cascades is defined by the ration between the γ_{LDI} and the Landau damping. In the contrary, in the case of smaller k_p the spectrum is much broader and it

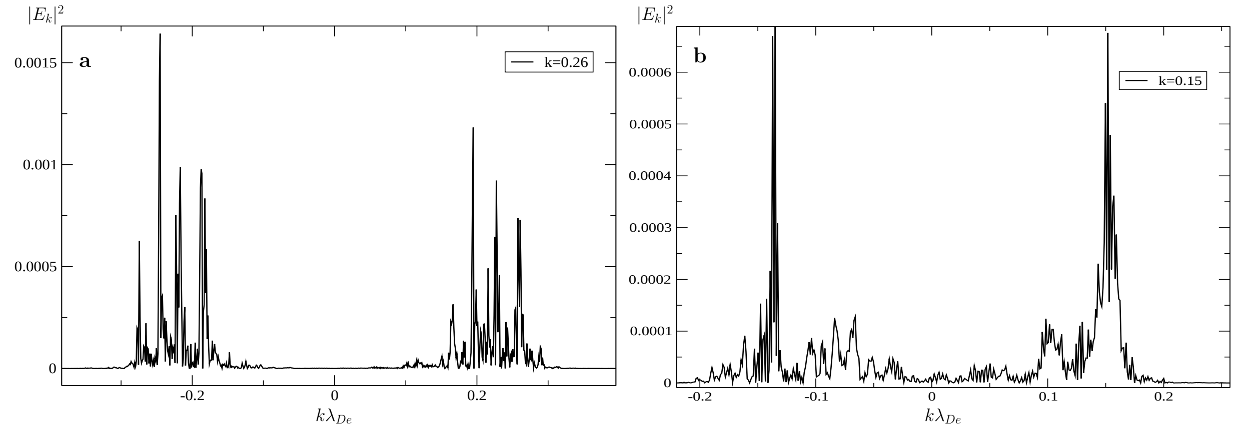

FIG. 10: Asymptotic EPW spectra ($\omega_{pe}t = 5400$) for $k_p\lambda_{De} = 0.26$ (a) and $k_p\lambda_{De} = 0.15$ (b). Other parameters are the same as in Fig. 9.

extends to a very long wavelengths. In this case the LDI cascade terminates by the collapse and the electrons are accelerated more efficiently. This can be seen in the asymptotic electron distribution function shown in Fig. 11. In the case of smaller k_p the electron energy distribution spreads to much higher energies and the total number of fast electrons and their effective temperature are also higher.

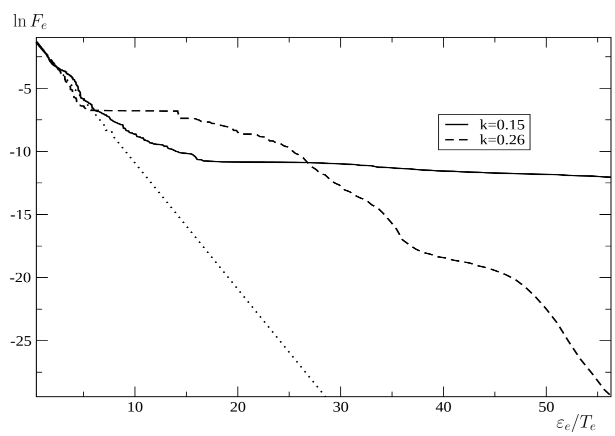

FIG. 11: Spatially averaged electron distribution function at the final time $\omega_{pe}t = 6000$ for the case of $k_p\lambda_{De} = 0.26$ (dashed line) and $k_p\lambda_{De} = 0.15$ (solid line). The dotted line corresponds to the initial electron distribution.

The quantitative characteristics of the EPW saturation are presented in Table I. The hot electron effective temperature was from the average slope of the electron distribution function in the diffusion region and the number of hot electrons was calculated from the relation $W_e = n_hT_h$

at the asymptotic stage. The maximum EPW energy increases with k_p and the hot electron temperature decreases. Moreover, T_h is much smaller than the energy of resonant electrons ε_{ph} in the case of small k_p and approaches it as k_p increases. The number of hot electrons demonstrates also a non-monotonic dependence on k_p, similar to that of W_e. The energy deposited in electrons in the asymptotic stage is always larger than that rest in the EPWs. Even in the long wavelength case where $k_p\lambda_{De} \lesssim 0.1$ and the quasi-linear supposed to play a minor role, W_e at the asymptotic stage is twice larger than W_E. This observation agrees with the conclusion of Ref. [14] where the quasi-linear-Zakharov model (QLZ) was compared with the solution to the ZAkharov equations and to the reduced PIC model. It was concluded that even in the case of a small driver wave number the Zakharov model overestimates the level of the EPW saturation while the QLZ model is in a good agreement with the full kinetic simulations. In our case, only in the case of a very large $k_p\lambda_{De} \simeq 0.3$, at the limit of validity of the quasi-linear theory the particle and wave energy become comparable. In the contrary, for the optimal $k_p \simeq k_*$ the wave energy is very small, about seven times smaller than the energy of hot electrons.

TABLE I: Characteristics of the EPW saturation in function of the driver wave number for the case of driver amplitude $\bar{S}_0 = 7.6 \times 10^{-3}$ and $\Delta L\, \lambda_{De} = 250$.

$k_p\lambda_{De}$	ε_{ph}/T_e	$\max W_E$	$W_{E\,sat}$	$W_{e\,sat}$	W_{tot}	T_h/T_e	n_h/n_0,%
0.12	34.7	0.17	0.012	0.025	0.037	10	0.25
0.15	22.2	0.18	0.01	0.035	0.036	9	0.39
0.18	15.4	0.21	0.007	0.048	0.055	9.5	0.50
0.22	10.3	0.24	0.014	0.028	0.042	8	0.35
0.24	8.7	0.26	0.016	0.023	0.03	7,5	0.30
0.26	7.4	0.32	0.021	0.018	0.036	7	0.26

It should be noted that the effect of complete saturation and decoupling of the driver is related to the periodic boundary conditions used in our calculations. Although it is not important for the waves – neither EPWs nor IAWs do not reach the boundaries during the run time – the fast electrons recirculate and establish a quasi-stationary asymptotic state after a thousand plasma wave periods. The effect of non-periodic boundary conditions will be discussed in the following section.

C. Effect of the particle losses from the interaction region

The loss term in the quasi-linear equation (3) was proposed in Ref. [15] in order to account in a 1D model the particle losses in the perpendicular plane. This term acts to restore the original distribution function, mimicking the absorbing boundary conditions, which replace heated electrons leaving the simulation domain with a Maxwellian distribution. Evidently the effect of convective losses depends on the ratio between the width on the excitation region ΔL and the effective loss length L which coincides with the simulation box length in out case. The effect of losses might be important is $\Delta L \simeq L$. Then the electron distribution function stays close to the Maxwellian distribution and the QLZ model gives the results very similar to the Zakharov model. In the contrary, in the case where $\Delta L \ll L$ does not effect strongly the saturation. This is demonstrated below for the reference set of parameters.

Figure 12 shows the temporal evolution of the spatially averaged EPW energy W_E and the electron energy W_e. The additional damping term has a negligible effect on the EPW energy, while the temporal evolution of the electron energy is modified at the time when the electrons become to leave the computational domain. At this time, the incoming electron distribution is replaced by the Maxwellian distribution, so the hot electron energy drops down.

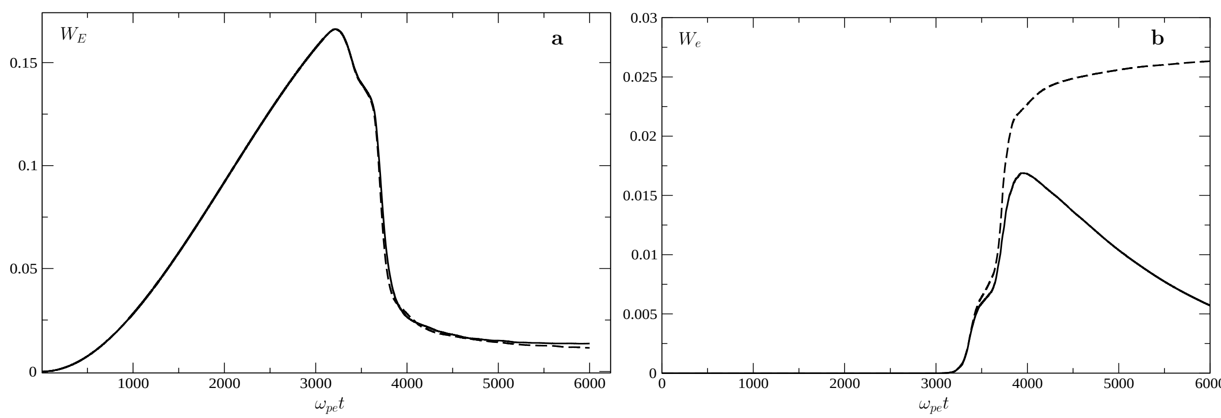

FIG. 12: Temporal evolution of the spatially averaged EPW energy W_E (a) and the electron energy W_e (b) with the additional damping term (solid line) and without it (dashed line) for the reference set of parameters.

Figure 13 shows the effect of perpendicular losses on the spatially averaged electron distribution function. In both cases the shape of the electron distribution function and the effective hot electron temperature are approximately the same while the number of hot electrons decreases. Therefore, the particle losses in the case of a narrow excitation region have a small effect on the residual

Landau damping and on the EPW saturation level.

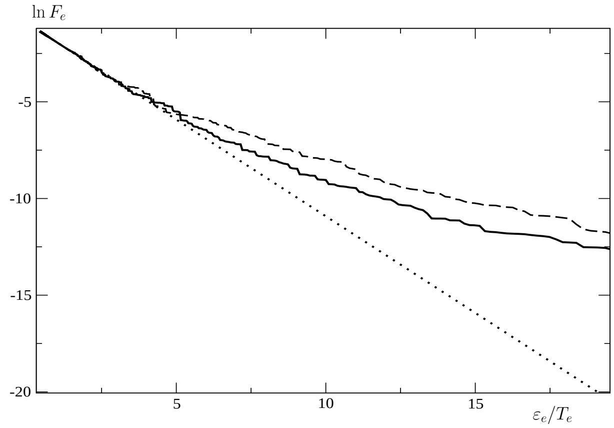

FIG. 13: Averaged electron distribution function at initial time (dotted line) and at the time of $\omega_{pe}t = 5000$, with the additional damping term (solid line) and without it (dashed line) for the reference set of parameters.

V. SUMMARY AND CONCLUSIONS

We have studied the temporal evolution and the saturation of a driven EPW wave due to the wave-wave and wave-particle interactions. The process has been considered within the QLZ model. Here the electron kinetic equation and the equation for the EPW are averaged over the electron plasma period. This strongly simplifies the numerical solution and allows one to consider the time evolution in the scale of a few thousand plasma periods.

The system was driven by a spatially localized monochromatic driver. The temporal evolution of the EPWs comprises several characteristic steps: (i) the excitation of the primary EPW, (ii) its nonlinear saturation due to the nonlinear frequency shift, (iii) a cascade of LDIs which generates long wavelength EPWs, (iv) the modulational instability of these EPWs and their collapse, (v) formation of short wavelength EPWs and their interaction with electrons, (vi) final saturation of the EPWs at a low level and their decoupling from the driver. In the case of a very short wavelength driver, $k_p\lambda_{De} > 0.18$, the LDI cascade terminates without collapse. Then the wave-particle interaction becomes less efficient and the number of hot electrons decreases.

Compared to the standard Zakharov model with a fixed electron distribution function we confirm the conclusion of Ref. [14] that the quasi-linear evolution plays an important role even in a case of long wavelength driver by decreasing the saturation level of the EPWs and producing a significant number of hot electrons. It was found that the effective temperature of hot electrons is significantly smaller than the kinetic energy of the resonant electrons. This is a consequence of the fact that the electron acceleration is due to their interaction with short wavelength EPWs, $k > k_p$. These waves are generated at the nonlinear stage of the evolution as a result of the LDI cascade and the collapse of the long wavelength EPWs.

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