# Devoir Maison BMS

1|2|3|4|5
4|4|3|4|4

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## Exercise 1

1)

| $P$ | $Q$ | $P \vee Q$ | $P \wedge Q$ | $(P \vee Q) \Rightarrow (P \wedge Q)$ |
|---|---|---|---|---|
| F | F | F | F | T |
| T | F | T | F | F |
| F | T | T | F | F |
| T | T | T | T | T |

2)

| $P$ | $Q$ | $P \Rightarrow Q$ | $Q \Rightarrow P$ | $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$ |
|---|---|---|---|---|
| F | F | T | T | T |
| T | F | F | T | F |
| F | T | T | F | F |
| T | T | T | T | T |

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3) Both propositions $(P \vee Q) \Rightarrow (P \wedge Q)$ and $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$ have the same truth table, therefore both propositions are equivalent. The second proposition, $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$, is also equivalent to $P \Leftrightarrow Q$, by definition. Therefore, $(P \vee Q) \Rightarrow (P \wedge Q)$ is equivalent to $P \Leftrightarrow Q$.

## Exercise 2

1) $f$ has a maximum if $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, f(x) \geq f(y)$ is true.

2) $f$ doesn't have a minimum if $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, f(x) > f(y)$ is true.

3) Let $y \in \mathbb{R}$, we can choose $x \in \mathbb{R}$ such that $x = y$
$\Rightarrow f(x) = f(y) \Rightarrow f(x) \leq f(y)$
Therefore $\forall y \in \mathbb{R}, \exists x \in \mathbb{R}, f(x) \leq f(y)$

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Exercise 3

1) $\sum_{k=1}^{1} k = 1$ and $\frac{1(1+1)}{2} = 1$

therefore $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ for $n=1$.

?????

Lets assume $\exists n \in \mathbb{N}^*$ such that $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ and show that it follows that $\sum_{k=1}^{n+1} k = \frac{(n+1)(n+2)}{2}$

$$\sum_{k=1}^{n+1} k = \sum_{k=1}^{n} k + n+1 = \frac{n(n+1)}{2} + n+1 = \frac{n(n+1)+2(n+1)}{2}$$

$= \frac{(n+2)(n+1)}{2}$ therefore $\left(\sum_{k=1}^{n} k = \frac{n(n+1)}{2}\right) \Rightarrow \left(\sum_{k=1}^{n+1} k = \frac{(n+1)(n+2)}{2}\right)$

In conclusion $\sum_{k=1}^{1} k = \frac{1(1+1)}{2}$ and $\left(\sum_{k=1}^{n} k = \frac{n(n+1)}{2}\right)$

$\Rightarrow \left(\sum_{k=1}^{n+1} k = \frac{(n+1)(n+2)}{2}\right)$ for all $n \in \mathbb{N}^*$,

therefore $\forall n \in \mathbb{N}^*$, $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

2) Let the property $P(n)$ be $\sum_{k=1}^{n} k^3 = \left(\sum_{k=1}^{n} k\right)^2$.

$\sum_{k=1}^{1} k^3 = 1^3 = 1$ and $\left(\sum_{k=1}^{1} k\right)^2 = 1^2 = 1$

therefore $P(1)$ is true.

Lets assume $\exists n \in \mathbb{N}^*$ such that $P(n)$ and show that it follows that $P(n+1)$ is true. For this we will use the result proved in the previous question, that is: $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

$$\sum_{k=1}^{n+1} k^3 = \sum_{k=1}^{n} k^3 + (n+1)^3 = \left(\sum_{k=1}^{n} k\right)^2 + (n+1)^3 = \left(\frac{n(n+1)}{2}\right)^2 + (n+1)^3$$

$$= \frac{n^2(n+1)^2}{4} + (n+1)^3 = \frac{n^2(n+1)^2 + 4(n+1)^3}{4} = \frac{(n+1)^2(n^2+4n+4)}{4}$$

$$=\frac{(n+1)^2(n+2)^2}{4}=\left(\frac{(n+1)(n+2)}{2}\right)^2=\left(\sum_{k=1}^{n+1}k\right)^2$$, therefore

$\forall n\in\mathbb{N}^*,\ P(n)\Rightarrow P(n+1)$

In conclusion $P(1)$ is true and $\forall n\in\mathbb{N}^*, P(n)\Rightarrow P(n+1)$

therefore $\forall n\in\mathbb{N}^*,\ \sum_{k=1}^{n}k^3=\left(\sum_{k=1}^{n}k\right)^2$.

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## Exercise 4

We have $x-1+|x-2|\geqslant|2x+4|$ (I).

$x-2\geqslant 0\Leftrightarrow x\geqslant 2$ and $2x+4\geqslant 0\Leftrightarrow x\geqslant -2$

If $x\in]-\infty;-2[$, (I) $\Leftrightarrow x-1-x+2\geqslant -2x-4$

$\Leftrightarrow 2x\geqslant -5\Leftrightarrow x\geqslant -\frac{5}{2}$ $\quad S=[-\frac{5}{2};-2[$

If $x\in[-2;2[$, (I) $\Leftrightarrow x-1-x+2\geqslant 2x+4$

$\Leftrightarrow -3\geqslant 2x\Leftrightarrow -\frac{3}{2}\geqslant x$ $\quad S=[-2;-\frac{3}{2}]$

If $x\in[2;+\infty[$, (I) $\Leftrightarrow x-1+x-2\geqslant 2x+4$

$\Leftrightarrow -3\geqslant 4$ $\quad S=\emptyset$

Therefore $S=[-\frac{5}{2};-\frac{3}{2}]$.

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## Exercise 5

1) $\begin{cases} x+2y-3z=-7 \\ 4x-2y+2z=14 \\ -3x+y-z=-9 \end{cases}$ $\quad L_2\to\frac{1}{2}L_2-2L_1$ $\quad L_3\to L_3+3L_1$

$\Leftrightarrow\begin{cases} x+2y-3z=-7 \\ -5y+7z=21 \\ 7y-10z=-30 \end{cases}$ $\quad L_3\to 5L_3+7L_2$

$\Leftrightarrow\begin{cases} x+2y-3z=-7 \\ -5y+7z=21 \\ -z=-3 \end{cases}\Leftrightarrow\begin{cases} x+2y-9=-7 \\ -5y+21=21 \\ z=3 \end{cases}\Leftrightarrow\begin{cases} x-9=-7 \\ y=0 \\ z=3 \end{cases}$

$\Leftrightarrow\begin{cases} x=2 \\ y=0 \\ z=3 \end{cases}$ $\quad S=\{(2,0,3)\}$

$$2)\begin{cases} x+2y-3z=-7 \\ 4x-2y+2z=14 \\ -5x \qquad +z=-5 \end{cases} \qquad L_1 \to L_1+L_2+L_3$$

$$\Leftrightarrow \begin{cases} 0=2 \\ 4x-2y+2z=14 \\ -5x \qquad +z=-5 \end{cases} \qquad S=\emptyset$$