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GLORIA Clémence
PM101 GROUPE A12

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DM

4TPM103 U (Bases mathématiques pour les sciences)

1	2	3	4	5
4	1.5	4	4	4

Exercice 1 :

1) $(P \vee Q) \rightarrow (P \wedge Q)$

P	Q	$(P \vee Q) \rightarrow (P \wedge Q)$
F	F	T
F	T	F
T	F	F
T	T	T

2) $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$(P \Rightarrow Q) \wedge (Q \Rightarrow P)$
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	T	T	T

3) We know $P \Leftrightarrow Q \Rightarrow (P \Rightarrow Q) \wedge (Q \Rightarrow P)$
If we compare table of $(P \vee Q) \rightarrow (P \wedge Q)$ and table of $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$, they are the same.
So we can deduce : $P \Leftrightarrow Q = (P \vee Q) \rightarrow (P \wedge Q)$. 4/4

Exercice 2 :

1) " f admits a maximum " : $\exists y \in \mathbb{R}, \forall x \in \mathbb{R}, f(x) \leq f(y)$ $\leq$

2) " f does not admit a minimum" : $\forall y \in \mathbb{R}, \exists x \in \mathbb{R}, f(x) \leq f(y)$ $<$

3) $P : \forall y \in \mathbb{R}, \exists x \in \mathbb{R}, f(x) \leq f(y)$ 1.5/4
$\neg P : \exists y \in \mathbb{R}, \forall x \in \mathbb{R}\ \neg(f(x) \leq f(y))$
$\neg P : \exists y \in \mathbb{R}, \forall x \in \mathbb{R}\ (f(x) \leq f(y))$

$\rightarrow$ $\neg P$ is false and P is true justify!!!

Exercice 3:

1) $\forall n \in \mathbb{N}^*: \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

Initialization: for $n=1 \rightarrow \sum_{k=1}^{1} k = 1 = \frac{1(1+1)}{2} = \frac{1\times 2}{2} = 1$

The proposition is true at rank 1.

Hypothese: Assume $\sum_{k=1}^{n} = \frac{n(n+1)}{2}$ true for rank n.
We now want to check for (n+1) rank.

Verification hypothesis: $\sum_{k=1}^{n+1} k = \frac{(n+1)((n+1)+1)}{2}$, $\forall n \in \mathbb{N}^*$

$$\begin{aligned}
\text{So}: \sum_{k=1}^{n+1} k &= \sum_{k=1}^{n} k + \sum_{k=n+1}^{n+1} k \\
&= \frac{n(n+1)}{2} + n + 1 \\
&= \frac{n(n+1) + 2(n+1)}{2} \\
&= \frac{n^2 + 3n + 2}{2} \\
&= \frac{(n+1)(n+2)}{2}
\end{aligned}$$

So we have $\frac{(n+1)((n+1)+1)}{2} = \frac{(n+1)(n+2)}{2}$ which verifies the proposition is true.

Conclusion: $\forall n \in \mathbb{N}^* \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ true.

2) $\forall n \in \mathbb{N}^*, \sum_{k=1}^{n} k^3 = \left(\sum_{k=1}^{n} k\right)^2$

Initialization: for $n=1 \rightarrow \sum_{k=1}^{1} k^3 = 1^3 = 1 = \left(\sum_{k=1}^{1} k\right)^2 = 1^2 = 1$

The proposition is true at rank 1.

Hypothese: Assume $\sum_{k=1}^{n} k^3 = \left(\sum_{k=1}^{n} k\right)^2$ = true for rank n.
We now want to check for rank (n+1).

Verification hypothesis: $\sum_{k}^{n+1} k^3 = \left(\sum_{k=1}^{n+1} k\right)^2 = \left(\sum_{k=1}^{n+1} k\right)^2$

$$\begin{aligned}
\text{So}: \sum_{k=1}^{n+1} k^3 &= \sum_{k=1}^{n} k^3 + \sum_{k=n+1}^{n+1} k^3 \\
&= \left(\frac{n(n+1)}{2}\right)^2 + (n+1)^3 \\
&= \frac{n^2(n+1)^2}{4} + \frac{4(n+1)^3}{4} \\
&= \frac{(n+1)^2 (n^2 + 4(n+1))}{4} \\
&= \frac{(n+1)^2 (n+2)^2}{4} \\
&= \left(\frac{(n+1)(n+2)}{2}\right)^2
\end{aligned}$$

So we have $\frac{(n+1)(n+2)}{2} = \left(\frac{(n+1)(n+2)}{2}\right)^2$ which verifies the proposition is true

Conclusion: $\forall n \in \mathbb{N}^*, \sum_{k=1}^{n} k^3 = \left(\sum_{k=1}^{n} k\right)^2$ true.

Exercice 4:

$$x - 1 + |x-2| \geq |2x+4|$$
$$\Leftrightarrow x + |x-2| - |2x+4| \geq 1$$

$\rightarrow$ 4 possible cases:

* $x + x - 2 - (2x+4) \geq 1$ with $x - 2 > 0$ and $2x + 4 \geq 0$
* $x - (x-2) - (2x+4) \geq 1$ with $x - 2 < 0$ and $2x + 4 \geq 0$
* $x + x - 2 - (-(2x+4)) \geq 1$ with $x - 2 \geq 0$ and $2x + 4 < 0$
* $x - (x-2) - (-(2x+4)) \geq 1$ with $x - 2 < 0$ and $2x + 4 < 0$

<u>First</u>: $x + x - 2 - (2x+4) \geq 1$ $\quad x - 2 \geq 0 \quad 2x + 4 \geq 0$

$\Leftrightarrow x + (-2) - 2x - 4 \geq 1$ $\quad x \geq 2 \quad x \geq -2$

$\Leftrightarrow -2 - 4 \geq 1$

$\Leftrightarrow -6 \geq 1 \rightarrow x \in \varnothing$

<u>Second</u>: $x - (x-2) - (2x+4) \geq 1$ $\quad x - 2 < 0 \quad 2x + 4 \geq 0$

$-2 - 2x \geq 1$ $\quad x < 2 \quad x \geq -2$

$-2x \geq 3$

$x \leq -\frac{3}{2}$

<u>Third</u>: $x + x - 2 - (-2x+4)) \geq 1$ $\quad x - 2 \geq 0 \quad 2x + 4 < 0$

$4x - 2 + 4 \geq 1$ $\quad x \geq 2 \quad x < -2$

$4x \geq -1$

$x \geq -\frac{1}{4}$

<u>Fourth</u>: $x - (x-2) - (-(2x+4)) \geq 1$ $\quad x - 2 < 0 \quad 2x + 4 < 0$

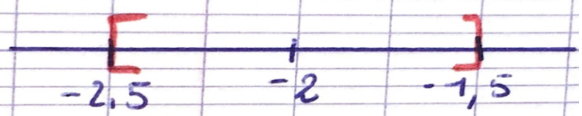
$2 + 2x + 4 \geq 1$ $\quad x < 2 \quad x < -2$

$2x \geq -5$

$x \geq -\frac{5}{2}$

Recap:

* $x \in \varnothing$, $x \geq 2$, $x \geq -2$ $\rightarrow x \in \varnothing$
* $x \leq -\frac{3}{2}$, $x < 2$, $x \geq -2$ $\rightarrow x \in [-2, -\frac{3}{2}]$
* $x \geq -\frac{1}{4}$, $x \geq 2$, $x < -2$ $\rightarrow x \in \varnothing$
* $x \geq -\frac{5}{2}$, $x < 2$, $x < -2$ $\rightarrow x\ [-\frac{5}{2}, -2]$



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The set of solution is: $x \in [\frac{5}{3}, -\frac{3}{2}]$

Exercice 5 :

1) $\begin{cases} x + 2y - 3z = -7 \\ 4x - 2y + 2z = 14 \\ -3x + y - z = -9 \end{cases} \overset{L_2 \to \frac{1}{2}L_2}{\iff} \begin{cases} x + 2y - 3z = -7 \\ 2x - y + z = 7 \\ -3x + y - z = -9 \end{cases} \overset{L_2 \to L_2 + L_3}{\iff}$

$L_2 \to L_2 + L_3$

$\begin{cases} x + 2y - 3z = -7 \\ 2x + (-3x) - y + y + z - z = 7 + (-9) \end{cases} \rightarrow \boxed{x = 2}$

$L_1 \to 2L_2$

$\begin{cases} x + (4x) + 2y + (-2y) - 3z + (2z) = -7 + 14 \\ -3x + y - z = -9 \end{cases} \rightarrow 5x - z = 7$

$\iff \boxed{z = 3}$

$2x - y + z = 7$

$\iff 2 \times 2 - y + 3 = 7$

$\iff 7 - y = 7$

$\iff \boxed{y = 0}$

Verification : $x + 2y - 3z = -7$

$\iff 2 + 2 \times 0 - 3 \times 3 = -7$

$\iff -7 = -7$

$\Rightarrow$ The set of solution is : $\{(2, 0, 3)\}$

2) $\begin{cases} x + 2y - 3z = -7 \\ 4x - 2y + 2z = 14 \\ -5x + z = -5 \end{cases} \overset{L_2 \to \frac{1}{2}L_2}{\iff} \begin{cases} x + 2y - 3z = 7 \\ 2x - y + z = 7 \\ -5x + z = -5 \end{cases} \rightarrow x = 1 + \frac{1}{5}z$

$\begin{cases} 1 + \frac{1}{5}z + 2y - 3z = -7 \\ 2\left(1 + \frac{1}{5}z\right) - y + z = 7 \end{cases} \iff \begin{cases} -10y - 14z = -40 \\ 2\left(1 + \frac{1}{5}z\right) - y + z = 7 \end{cases} \iff \begin{cases} -10y - 14z = -40 \\ -5y + 7z = 25 \end{cases}$

$\iff \begin{cases} 5y - 7z = -20 \\ -5y + 7z = 25 \end{cases} \iff 5y - 7z - 5y + 7z = -20 + 25$

$\iff -7z + 7z = -20 + 25$

$\iff \boxed{0 = 5}$ impossible

$\rightarrow$ The set of solution is : $(x, y, z) \in \emptyset$