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Pour le 10/11/20

# DM BMS

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 4 | 2.5 | 4 | 4 | 4 |

18.5/20

## Exercice 1

1) $(P \vee Q) \Rightarrow (P \wedge Q)$

| P | Q | $P \vee Q$ | $P \wedge Q$ | $(P \vee Q) \Rightarrow (P \wedge Q)$ |
|---|---|---|---|---|
| F | F | F | F | T |
| F | T | T | F | F |
| T | T | T | T | T |
| T | F | T | F | F |

2)3) $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$

| P | Q | $P \Rightarrow Q$ | $Q \Rightarrow P$ | $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$ | $P \Leftrightarrow Q$ |
|---|---|---|---|---|---|
| F | F | T | T | T | T |
| F | T | T | F | F | F |
| T | T | T | T | T | T |
| T | F | F | T | F | F |

As we can see above, $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$ and $P \Leftrightarrow Q$ have the same truth tables. They are thus equivalent.

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## Exercice 2

1) "f admits a maximum" : $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, f(x) \geq f(y)$

2) "f doesn't admit a minimum" : $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, f(x) > f(y)$

3) P : "$\forall y \in \mathbb{R}, \exists x \in \mathbb{R}, f(x) \leq f(y)$"

Let us show that this proposal is always true, $\forall f$ of $\mathbb{R}$ in $\mathbb{R}$.

In order to do that, we will demonstrate it by the absurd.

Let's admit nonP : " $\exists y \in \mathbb{R}, \forall x \in \mathbb{R}, f(x) > f(y)$ " $\forall f$ of $\mathbb{R}$ in $\mathbb{R}$.

Let's take $f : x \mapsto 2x$ as function $f$., and $x = -2y$ ???????

$f(x) > f(y)$

$2x > 2y$

$2x - 2y > 2y$

$-4y > 2y$ $\Rightarrow$ IMPOSSIBLE

nonP is not true for all functions of $\mathbb{R}$ in $\mathbb{R}$, so nonP is false.

P : " $\forall y \in \mathbb{R}, \exists x \in \mathbb{R}, f(x) \leq f(y)$ " is thus verified. **2.5/4**

Exercice 3

1) Let us show that $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$, $\forall n \in \mathbb{N}^*$.

$P(n)$ : " $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ ".

- initialisation : $P(n=1)$

$\sum_{k=1}^{1} k = 1$ $\qquad \frac{1(1+1)}{2} = \frac{2}{2} = 1$ $\qquad P(n=1)$ is true.

- Hérédité :

Let us prove that $\forall n \geq 1$, $P(n) \Rightarrow P(n+1)$.

We assume $P(n)$ : " $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ "

We want to prove $P(n+1)$ : " $\sum_{k=1}^{n+1} k = \frac{n+1((n+1)+1)}{2}$ ", $\forall n \in \mathbb{N}^*$

$\sum_{k=1}^{n+1} k = \sum_{k=1}^{n} k + (n+1)$. Thanks to $P(n)$, we can say $\sum_{k=1}^{n+1} k = \frac{n(n+1)}{2} + (n+1)$

$\Leftrightarrow \frac{n(n+1)}{2} + \frac{2(n+1)}{2}$

$\Leftrightarrow \frac{n(n+1) + 2(n+1)}{2}$

$\Leftrightarrow \frac{n^2 + n + 2n + 2}{2}$

$\Leftrightarrow \frac{n^2 + 3n + 2}{2}$

$\Leftrightarrow \frac{(n+1)(n+2)}{2}$

$\Leftrightarrow \frac{n+1((n+1)+1)}{2}$

- Conclusion : $\forall n \in \mathbb{N}^*$, $P(n+1)$ is true, thus $P(n)$ : " $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ " is true as well.

2) We want to prove that, $\forall n \in \mathbb{N}^*$, $P(n)$ : " $\sum_{k=1}^{n} k^3 = \left(\sum_{k=1}^{n} k\right)^2$ " true.

- <u>initialisation</u> : $P(n=1)$

$\sum_{k=1}^{1} k^3 = 1^3 = 1$ $\qquad$ $\left(\sum_{k=1}^{1} k\right)^2 = \left(\sum_{k=1}^{1} k\right) \times \left(\sum_{k=1}^{1} k\right) = 1 \times 1 = 1$

$P(n=1)$ is true

- <u>Hérédité</u> :

Let us prove that $\forall n \geq 1$, $P(n) \Rightarrow P(n+1)$

We thus assume $P(n)$ : " $\sum_{k=1}^{n} k^3 = \left(\sum_{k=1}^{n} k\right)^2$ " true

We want to prove that $P(n+1)$ : " $\sum_{k=1}^{n+1} k^3 = \left(\sum_{k=1}^{n+1} k\right)^2$ " is true as well.

$\sum_{k=1}^{n+1} k^3 = \sum_{k=1}^{n} k^3 + (n+1)^3 = \left(\sum_{k=1}^{n} k\right)^2 + (n+1)^3$ (→ thanks to $P(n)$)

$\Leftrightarrow \left(\frac{n(n+1)}{2}\right)^2 + (n+1)^3$ (→ thanks to q° 1))

identité remarquable

$\Leftrightarrow \frac{n^2(n+1)^2 + 4(n+1)^3}{4} = \frac{(n+1)^2(n^2+4(n+1))}{4} = \frac{(n+1)^2(n^2+4n+4)}{4}$

$\Leftrightarrow \frac{(n+1)^2(n+2)^2}{4} = \left(\frac{(n+1)(n+2)}{2}\right)^2$

We have proven $\sum_{k=1}^{n+1} k^3 = \left(\frac{(n+1)(n+2)}{2}\right)^2$.

Like that, $\sum_{k=1}^{n+1} k = \frac{(n+1)(n+2)}{2}$ and $\left(\sum_{k=1}^{n+1} k\right)^2 = \left(\frac{(n+1)(n+2)}{2}\right)^2$

We thus obtain $\sum_{k=1}^{n+1} k^3 = \left(\frac{(n+1)(n+2)}{2}\right)^2 = \left(\sum_{k=1}^{n+1} k\right)^2$

→ $P(n+1)$ is true.

- <u>Conclusion</u> : $\forall n \in \mathbb{N}^*$, $P(n+1)$ is true. Thus $P(n)$ : " $\sum_{k=1}^{n} k^3 = \left(\sum_{k=1}^{n} k\right)^2$ " is true as well.

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## <u>Exercice 4</u>

1) Let us resolve $x - 1 + |x-2| \geq |2x+4|$ in $\mathbb{R}$ :

$|x-2| = \begin{cases} -x+2 & \text{if } x < 2 \\ x-2 & \text{if } x \geq 2 \end{cases}$ $\qquad$ $|2x+4| = \begin{cases} -2x-4 & \text{if } x \leq -2 \\ 2x+4 & \text{if } x \geq -2 \end{cases}$


We have to study 3 cases :

① if $x \in ]-\infty ; -2]$ :

$x - 1 + (-x+2) \geq -2x - 4$

$x - 1 - x + 2 \geq -2x - 4$

$1 \geq -2x - 4$

$5 \geq -2x$

$x \geq \frac{-5}{2}$ The set of solution is $[\frac{-5}{2} ; -2]$

② if $x \in [-2, 2]$ :

$x-1+(-x+2) \geq 2x+4$

$x-1-x+2 \geq 2x+4$

$1 \geq 2x+4$

$-3 \geq 2x$

$x \leq -\frac{3}{2}$

The set of solution is $[-2; -\frac{3}{2}]$

③ if $x \in [2; +\infty[$

$x-1+(x-2) \geq 2x+4$

$x-1+x-2 \geq 2x+4$

$2x-3 \geq 2x+4$

$x-3 \geq x+4$

$0 \geq 7$ $\Rightarrow$ IMPOSSIBLE

There is no solution on this interval

Conclusion : The total set of solution is $[-\frac{5}{2}; 2] \cup [-2; -3/2] \cup \emptyset$, that is to say $S = [-\frac{5}{2}; -\frac{3}{2}]$

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Exercice 5

1) $\begin{cases} x+2y-3z=-7 \\ 4x-2y+2z=14 \\ -3x+y-z=-9 \end{cases}$ $L_3 \Leftrightarrow L_2$ $\begin{cases} x+2y-3z=-7 \\ -3x+y-z=-9 \\ 4x-2y+2z=14 \end{cases}$ $L_3=L_3-4L_1$ $\begin{cases} x+2y-3z=-7 \\ -3x+y-z=-9 \\ 10y-14z=-42 \end{cases}$ $L_2 \Leftrightarrow L_3$

$\begin{cases} x+2y-3z=-7 \\ 10y-14z=-42 \\ -3x+y-z=-9 \end{cases}$ $L_3=L_3+3L_1$ $\begin{cases} x+2y-3z=-7 \\ 10y-14z=-42 \\ -5y-10z=-30 \end{cases}$ $L_3=2L_3+L_2$ $\begin{cases} x+2y-3z=-7 \\ 10y-14z=-42 \\ -34z=-102 \end{cases}$

$\rightarrow$ $-34z=-102 \Leftrightarrow z=\frac{102}{34} \Leftrightarrow \boxed{z=3}$

$\rightarrow$ $10y-14\times3=-42 \Leftrightarrow 10y=0 \Leftrightarrow \boxed{y=0}$

$\rightarrow$ $x+2\times0-3\times3=-7 \Leftrightarrow x-9=-7 \Leftrightarrow \boxed{x=2}$

The set of solution is $\{(2,0,3)\}$

2) $\begin{cases} x+2y-3z=-7 \\ 4x+2y+2z=14 \\ -5x+z=-5 \end{cases}$ $L_2=L_2-4L_1$ $\begin{cases} x+2y-3z=-7 \\ -10y+14z=42 \\ -5x+z=-5 \end{cases}$ $L_3=L_3-(-L_2)$ $\begin{cases} x+2y-3z=-7 \\ -10y+14z=42 \\ \boxed{0=2} \end{cases}$

$\Rightarrow$ IMPOSSIBLE : The system has no solution, the set of solution is $\emptyset$.

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