(1) Résoudre dans C l'équation w^2 = −5 − 12i.
(2) Résoudre dans C l'équation z^2 + iz + 1 + 3i = 0.

(1) $w^2 = -5-12i$

$w = a + bi$

$w^2 = a^2 - b^2 + 2abi = -5-12i$

$$\begin{cases} a^2 - b^2 = -5 & L_1 \\ 2ab = -12 & L_2 \\ a^2 + b^2 = 13 & L_3 \end{cases}$$

A third equation:

$|w|^2 = a^2 + b^2$ $\quad |w|^2 = |w^2| = |-5-12i|$

$= \sqrt{(-5)^2 + (-12)^2} = \sqrt{169} = 13$

$L_1 + L_3$: $2a^2 = 13 - 5 = 8$ $\quad a^2 = 4$ $\quad a = \pm 2$

$L_3 - L_1$: $2b^2 = 13 + 5 = 18$ $\quad b^2 = 9$ $\quad b = \pm 3$

$a = \pm 2$ $\quad b = \pm 3$

L_2: $a = 2$ $\quad b = -3$
or
$a = -2$ $\quad b = 3$

$w = 2 - 3i$ or $-2 + 3i$

The set of solutions is $\{2-3i, -2+3i\}$

z^2 + iz + 1 + 3i = 0.

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$$z^2 + iz + 1 + 3i = 0$$

$$\Delta = i^2 - 4(1+3i) = -1 - 4 - 12i$$
$$= -5 - 12i$$

Let us find δ such that $\delta^2 = \Delta$

Question 1 tells us that

$\delta = 2 - 3i$ would do.

$$z_1 = \frac{-i-\delta}{2} = \frac{-i-2+3i}{2} = -1+i$$

$$z_2 = \frac{-i+\delta}{2} = \frac{-i+2-3i}{2} = 1-2i$$

$az^2+bz+c=0$ to solve in $z \in \mathbb{C}$

1. Calculate $\Delta = b^2-4ac$
2. If $\Delta \neq 0$, find δ such that $\delta^2 = \Delta$
3. the roots are $z_1 = \frac{-b-\delta}{2a}$ $z_2 = \frac{-b+\delta}{2a}$
4. If $\Delta = 0$, the (double) root is $z = \frac{-b}{2a}$

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2 special cases:

1. $\Delta \in \mathbb{R}$ $\Delta > 0$
   then $\delta = \sqrt{\Delta}$
2. $\Delta \in \mathbb{R}$ $\Delta < 0$
   then $\delta = i\sqrt{|\Delta|}$

82 (1) $e^{\theta i} = \cos\theta + i\sin\theta$

$e^{-\theta i} = \cos\theta - i\sin\theta$

$e^{\theta i} + e^{-\theta i} = 2\cos\theta$

$\cos\theta = \frac{1}{2}\left(e^{\theta i} + e^{-\theta i}\right)$

(2) $\left(e^{\theta i} + e^{-\theta i}\right)^5$ using Newton's binomial formula

$(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

use this with $x = e^{\theta i}$ $y = e^{-\theta i}$

$\left(e^{\theta i} + e^{-\theta i}\right)^5 = e^{5\theta i} + 5e^{3\theta i} + 10e^{\theta i} + 10e^{-\theta i} + 5e^{-3\theta i} + e^{-5\theta i}$

(3) $\left(e^{\theta i} + e^{-\theta i}\right)^5 = e^{5\theta i} + e^{-5\theta i}$

$+ 5e^{3\theta i} + 5e^{-3\theta i} + 10e^{\theta i} + 10e^{-\theta i} =$

$= 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$

$\quad\quad -\theta i)\big)^5$

$$(\cos\theta)^5 = \left(\frac{1}{2}\left(e^{\theta i} + e^{-\theta i}\right)\right)^5 =$$

$$= \frac{1}{2^5}\left(e^{\theta i} + e^{-\theta i}\right)^5 =$$

$$= \frac{1}{32}\left(2\cos 5\theta + 10\cos 3\theta + 20\cos\theta\right)$$

$$= \frac{1}{16}\cos 5\theta + \frac{5}{16}\cos 3\theta + \frac{5}{8}\cos\theta$$

0	1
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1

$$(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

$$\begin{cases} x + y = 3 \\ t^2 x + y = 3t \end{cases}$$ find solution depending on the parameter t

$L_2 \to L_2 - t^2 L_1$

$$\begin{cases} x + y = 3 \\ (1-t^2) y = 3(t-t^2) \end{cases}$$

Solving the system depends on whether or not $1-t^2 = 0$

$1 - t^2 = 0 \iff t \in \{1, -1\}$

3 cases: $t \notin \{1,-1\}$; $t = 1$; $t = -1$.

The case $t \notin \{1,-1\}$

In this case $1-t^2 \neq 0$, and we solve the system in the usual fashion

$$y = \frac{3(t-t^2)}{1-t^2} = \frac{3t(1-t)}{(1-t)(1+t)} = \frac{3t}{1+t}$$

$$y = \frac{[illegible]}{1 - t^2} = \frac{[illegible]}{(1-t)(1+t)} = \frac{3t}{1+t}$$

$$x + \frac{3t}{1+t} = 3$$

$$x = 3 - \frac{3t}{1+t} = \frac{3(1+t) - 3t}{1+t} = \frac{3}{1+t}$$

The set of solutions is

$$\left\{ \left(\frac{3}{1+t}, \frac{3t}{1+t} \right) \right\}$$

The case $t = 1$

$$\begin{cases} x + y = 3 \\ 0y = 0 \end{cases}$$

Second equation can be eliminated

$x + y = 3 \qquad x = 3 - y$

The set of solutions is

$$\{ (3-y, y) : y \in \mathbb{R} \}$$

The case $t = -1$

$$\begin{cases} x + y = 3 \\ 0y = -6 \end{cases} \quad \text{impossible}$$

$\{ \; 0y = -6$ impossible

The set of solutions is $\emptyset$

The set of solutions is

$\left\{\left(\frac{3}{1+t}, \frac{3t}{1+t}\right)\right\}$ when $t \notin \{1, -1\}$

$\{(3-y, y) : y \in \mathbb{R}\}$ when $t = 1$

$\emptyset$ when $t = -1$