112, 118, 119

112 : objective is solving $-4x^3+3x+\frac{1}{2}=0$

① $f(x)=-4x^3+3x+\frac{1}{2}$

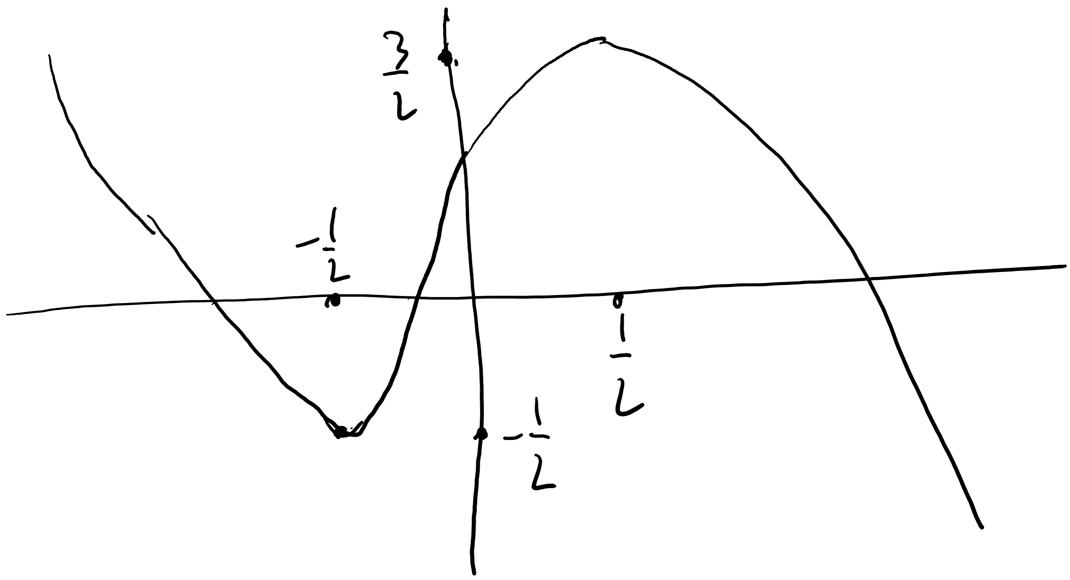
$f'(x)=-12x^2+3$

$f'(x)=0$ $x=\frac{1}{2}$ and $x=-\frac{1}{2}$

$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} 4x^3\left(-1+\frac{3}{4x^2}+\frac{1}{8x^3}\right)=-\infty$

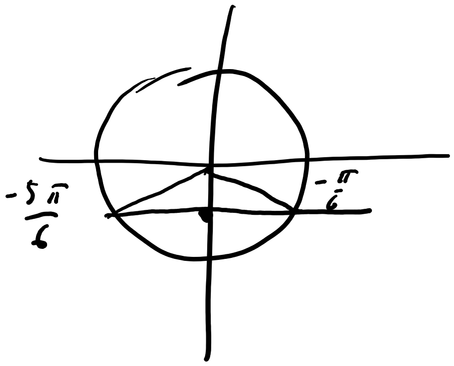
$\lim_{x\to-\infty} f(x) = +\infty$ $f(-\frac{1}{2})=-\frac{1}{2}$ $f(\frac{1}{2})=\frac{3}{2}$

x	$-\infty$	$]-\infty;-\frac{1}{2}[$	$-\frac{1}{2}$	$]-\frac{1}{2};\frac{1}{2}[$	$\frac{1}{2}$	$]\frac{1}{2};+\infty[$	$+\infty$
$f'(x)$		$-$	0	+	0	$-$	
$f(x)$	$+\infty$	↘	min $-\frac{1}{2}$	↗	max $\frac{3}{2}$	↘	$-\infty$



② (looks unrelated!)

Solve $\sin(3x) = -\frac{1}{2}$ on $[0; 2\pi]$



$3x = -\frac{\pi}{6} + 2\pi k$ or $-\frac{5\pi}{6} + 2\pi k$ $k \in \mathbb{Z}$

$x = -\frac{\pi}{18} + \frac{2}{3}\pi k$ or $-\frac{5\pi}{18} + \frac{2}{3}\pi k$

need to find $k \in \mathbb{Z}$ such that

$-\frac{\pi}{18} + \frac{2}{3}\pi k \in [0; 2\pi]$ $(*)$

need to find $k \in \mathbb{Z}$ such that

$-\frac{5\pi}{18} + \frac{2}{3}\pi k \in [0; 2\pi]$ $(**)$

$(*)$ $0 \le -\frac{\pi}{18} + \frac{2}{3}\pi k \le 2\pi$ divide by π

$0 \le -\frac{1}{18} + \frac{2}{3}k \le 2$ mult by $\frac{3}{2}$

$0 \le -\frac{1}{12} + k \le 3$ add $\frac{1}{12}$

$\frac{1}{12} \le k \le 3 + \frac{1}{12}$

Since $k \in \mathbb{Z}$ the solutions are

$$k \in \{1, 2, 3\}$$

$$(**) \quad 0 \leq \frac{-5\pi}{18} + \frac{2}{3}\pi k \leq 2\pi$$

$$0 \leq -\frac{5}{18} + \frac{2}{3} k \leq 2$$

$$0 \leq -\frac{5}{12} + k \leq 3$$

$$\frac{5}{12} \leq k \leq 3 + \frac{5}{12}$$

$$k \in \{1, 2, 3\}$$

$-\frac{\pi}{18} + \frac{2}{3}\pi k \qquad k \in \{1,2,3\}$ are

$$-\frac{\pi}{18} + \frac{2}{3}\pi = \frac{11}{18}\pi$$

$$-\frac{\pi}{18} + \frac{4}{3}\pi = \frac{23}{18}\pi$$

$$-\frac{\pi}{18} + 2\pi = \frac{35}{18}\pi$$

$-\frac{5\pi}{18} + \frac{2}{3}\pi k \quad , \quad k \in \{1,2,3\}$ are

$$\frac{7\pi}{18}, \quad \frac{19\pi}{18}, \quad \frac{31\pi}{18}$$

The set of solutions is

$$\left\{ \frac{7\pi}{18}, \frac{11\pi}{18}, \frac{19\pi}{18}, \frac{23\pi}{18}, \frac{31\pi}{18}, \frac{35\pi}{18} \right\}$$

3 Show that $\sin(3x) = 3\sin x - 4(\sin x)^3$

Use Euler formula

$e^{xi} = \cos x + i \sin x \qquad (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$e^{3xi} = \cos(3x) + i\sin(3x)$

$e^{3xi} = (\cos x + i \sin x)^3 =$

$= (\cos x)^3 + 3i(\cos x)^2 \sin x - 3\cos x(\sin x)^2 - i(\sin x)^3$

$= (\cos x)^3 - 3\cos x(\sin x)^2 + i\left(3(\cos x)^2 \sin x - (\sin x)^3\right)$

$\cos(3x) =$ ↓

$\sin(3x) = 3(\cos x)^2 \sin x - (\sin x)^3 \qquad (\cos x)^2 + (\sin x)^2 = 1$

$= 3\left(1 - (\sin x)^2\right)\sin x - (\sin x)^2$

$= 3\sin x - 4(\sin x)^3$

1 December, 2020 09:16

(4) Solve $f(x)=0$

$$-4x^3+3x=-\frac{1}{2} \qquad (*)$$

from what we know, we can solve

$$-4(\sin t)^3+3\sin t=-\frac{1}{2} \quad \text{on } [0;2\pi]$$

(3) $\Rightarrow$ left-hand side is $\sin(3t)$

$$\sin(3t)=-\frac{1}{2} \quad \text{on } [0;2\pi]$$

(2) $\Rightarrow$ $t\in\left\{\frac{7\pi}{18},\frac{11\pi}{18},\frac{19\pi}{18},\frac{23\pi}{18},\frac{31\pi}{18},\frac{35\pi}{18}\right\}$

$x=\sin\frac{7\pi}{18}$ then x satisfies $(*)$

Similarly for $x=\sin\frac{11\pi}{18},\dots,\sin\frac{35\pi}{18}$

We found 6 solutions of $(*)$!!

Actually, there are only 3 solutions

$$\frac{7\pi}{18}+\frac{11\pi}{18}=\pi$$

$$\sin\frac{7\pi}{18}=\sin\frac{11\pi}{18}$$

$19\pi \quad 35\pi \quad 54\pi \quad = \quad 3$

$$\frac{19\pi}{18} + \frac{35\pi}{18} = \frac{54\pi}{18} = \pi + 2\pi$$

$$\sin\frac{19\pi}{18} = \sin\frac{35\pi}{18}$$

$$\frac{23\pi}{18} + \frac{31\pi}{18} = \pi + 2\pi$$

$$\sin\frac{23\pi}{18} = \sin\frac{31\pi}{18}$$

$$\boxed{\sin(\pi - x) = \sin x}$$

The set of solutions of $(*)$

is $\left\{\sin\frac{7\pi}{18}, \sin\frac{19\pi}{18}, \sin\frac{23\pi}{18}\right\}$

118

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for $x \in [-1; 1]$

$$\arcsin x + \arccos x = \frac{\pi}{2}$$

$f : I \to J$ bijection $g : J \to I$

inverse function for f

$$f \circ g(x) = x \qquad x \in J$$

$$f'(g(x)) \cdot g'(x) = 1$$

$$g'(x) = \frac{1}{f'(g(x))}$$

Will use this in the following set up

$$I = [-\tfrac{\pi}{2}, \tfrac{\pi}{2}] \qquad J = [-1; 1]$$

$$f(x) = \sin x \qquad g(x) = \arcsin x$$

$$\arcsin' x = \frac{1}{\quad}$$

$$\arcsin' x = \frac{1}{\sin'(\arcsin x)}$$

$$= \frac{1}{\cos(\arcsin x)}$$

$$(\cos u)^2 + (\sin u)^2 = 1$$

$$|\cos u| = \sqrt{1-(\sin u)^2}$$

$$|\cos(\arcsin x)| = \sqrt{1-(\sin(\arcsin x))^2}$$

$$= \sqrt{1-x^2}$$

$$\arcsin x \in \left[-\frac{\pi}{2} ; \frac{\pi}{2}\right]$$

cos is positive on $\left[-\frac{\pi}{2} ; \frac{\pi}{2}\right]$

$$\cos(\arcsin x) = \sqrt{1-x^2}$$

In the same fashion:

$$\sin(\arccos x) = \sqrt{1-x^2}$$

$$\boxed{\arcsin' x = \frac{1}{\sqrt{1-x^2}} \qquad x \in]-1;1[}$$

$$\arccos' x = \frac{1}{}$$

$$\cos'(\arccos x)$$

$$= \frac{-1}{\sin(\arccos x)} = \frac{-1}{\sqrt{1-x^2}}$$

Return to 118

Want to prove

$\arcsin x + \arccos x = \frac{\pi}{2}$

$f(x) = \arcsin x + \arccos x \quad x \in [-1; 1]$

$f'(x) = \frac{1}{\sqrt{1-x^2}} + \frac{-1}{\sqrt{1-x^2}} = 0 \quad x \in]-1; 1[$

f is constant on $]-1; 1[$

To find the constant, we have to find one value of f

Let us find $f(0)$

$f(0) = \arcsin 0 + \arccos 0 = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

We proved

$f(x) = \frac{\pi}{2} \quad x \in]-1; 1[$

Break until 10h00

$f(-1) = \arcsin(-1) + \arccos(-1) = -\frac{\pi}{2} + \pi = \frac{\pi}{2}$

$f(1) = \arcsin(1) + \arccos(1) = \frac{\pi}{2} + 0 = \frac{\pi}{2}$

$f(x) = \frac{\pi}{2}$ for $x \in [-1; 1]$

$f(\frac{1}{2}) = \arcsin(\frac{1}{2}) + \arccos(\frac{1}{2}) =$

$= \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{2}$

1 December 2020 10:02

Ex 1.19

$f(x) = \arcsin(\sqrt{x})$

arc sin defined on $[-1;1]$

$\sqrt{x}$ defined on $[0;+\infty[$

$f(x)$ defined if $\sqrt{x} \in [-1;1]$

$x \in [0;1]$

The domain of definition of $f(x)$ is $[0;1]$

$f'(x) = \arcsin'(\sqrt{x}) \cdot (\sqrt{x})' =$

$$= \frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x(1-x)}}$$

f is derivable on $]0;1[$

$$h(x) = \arctan(x^2 - 1)$$

defined on $\mathbb{R}$

$$h'(x) = \arctan'(x^2-1) \cdot (x^2-1)' =$$

$$= \frac{1}{1+(x^2-1)^2} \cdot 2x = \frac{2x}{2 + 2x^2 + x^4}$$

11.9 (2)

$g(x) = \arctan x + \arctan \frac{1}{x}$

defined on $\mathbb{R} \setminus \{0\} =$

$]-\infty; 0[\cup]0; +\infty[$

$$g'(x) = \arctan' x + \arctan'\left(\frac{1}{x}\right) \cdot \left(\frac{1}{x}\right)'$$

$$= \frac{1}{1+x^2} + \frac{1}{1+\left(\frac{1}{x}\right)^2} \cdot \frac{-1}{x^2} =$$

$$= \frac{1}{1+x^2} + \frac{-1}{x^2+1} = 0$$

g is constan on $]-\infty; 0[$ and

on $]0; +\infty[$

but the constants can be distinct!!

$g(1) = \arctan 1 + \arctan 1 = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

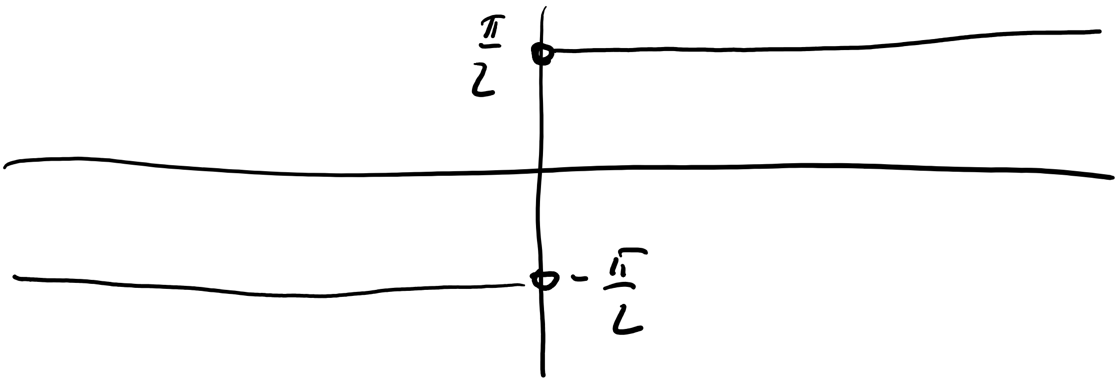
$g(x) = \frac{\pi}{2}$ for $x \in]0; +\infty[$

$$g(-1) = \arctan(-1) + \arctan(-1) =$$

$$= -\frac{\pi}{4} - \frac{\pi}{4} = -\frac{\pi}{2}$$

$g(x) = -\frac{\pi}{2}$ for $x \in]-\infty; 0[$

$$g(x) = \begin{cases} -\frac{\pi}{2}, & x \in]-\infty; 0[\\ \frac{\pi}{2}, & x \in]0; +\infty[\end{cases}$$



Ex 124, 125

124

$$\int x^a dx = \frac{x^{a+1}}{a+1} + C, \quad a \neq -1$$

$$\int x^{-1} dx = \ln x + C$$

$$\int x^3 dx = \frac{x^4}{4} + C$$

$$\int \frac{dx}{x^2} = \int x^{-2} dx = \frac{x^{-1}}{-1} + C$$

$$= -\frac{1}{x} + C$$

$$\int \frac{dx}{\sqrt{x}} = \int x^{-\frac{1}{2}} dx = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= 2\sqrt{x} + C$$

$$\int \frac{dx}{x\sqrt{x}} = \int x^{-\frac{3}{2}} dx = \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + C$$

$$= \frac{-2}{\ } + C$$

$$= \frac{-2}{\sqrt{x}} + C$$

$$\int \sin x \, dx = -\cos x + C$$

125 1

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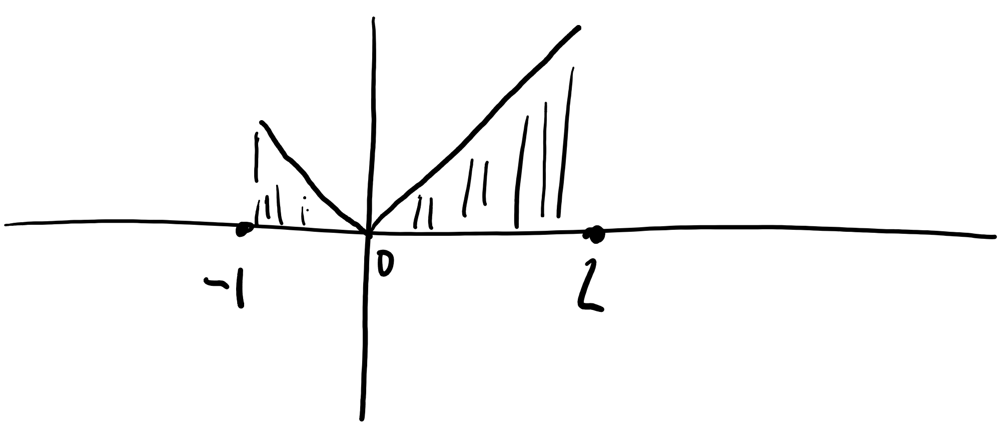
$$\int_{-1}^{2} |x|\,dx = \qquad \Big| \; |x| = \begin{cases} x, & x \in [0; +\infty[\\ -x, & x \in]-\infty; 0] \end{cases}$$

$$= \int_{-1}^{0} |x|\,dx + \int_{0}^{2} |x|\,dx =$$

$$= \int_{-1}^{0} -x\,dx + \int_{0}^{2} x\,dx =$$

$$= -\left[\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{2} =$$

$$= -\left(0 - \frac{1}{2}\right) + (2-0) = \frac{5}{2}$$



Homework 120

125 finish, 126

$|x|$ $\left(\frac{1}{2}x|x|\right)' = |x|$ true,

but I do not like

to use this