Title: Randomness and non-randomness in Piatetski-Shapiro sequences

Abstract: The sequences $\mathcal{N}^{(c)} = (\lfloor n^c \rfloor)_{n\in\mathbb{N}}$, where $c > 1$ is a real number which is not an integer, have been introduced in 1933 by B. I. Segal who studied their additive properties: there exists an integer $g(c)$ such that every integer is a sum of at most $g(c)$ elements from $\mathcal{N}^{(c)}$. On that respect they behave as regular polynomial sequences. Since at first glance they have no special arithmetic structure, one may expect that the sequence $\mathcal{N}^{(c)}$ contains infinitely many primes and even that the number of n up to x for which $\lfloor n^c \rfloor$ is prime is equivalent to $x/c\log x$: this was proved in 1953 for $c < 12/11$ by I. I. Piatetski-Shapiro.

A way to measure the complexity of a sequence $\mathcal{A}$ is to consider a map $\mathbf{u}$ from $\mathbb{N}$ to a finite set (without loss of generality, we may assume that $\mathbf{u}$ takes its values in $S_m = \{0, 1, \dots, m-1\}$) and to consider the distribution of the k-blocks $(\mathbf{u}(a_{n+1}), \mathbf{u}(a_{n+2}), \dots, \mathbf{u}(a_{n+k}))$ when n varies. In the talk, I'll report on recent results (involving also M. Drmota, C. Müllner, J. F. Morgenbesser and L. Spiegelhoffer) when $\mathcal{A}$ is a sequence $\mathcal{N}^{(c)}$ and when $\mathbf{u}$ is either the Thue-Morse sequence ($\mathbf{u}(n)$ is then the parity of the numbers of ones in the dyadic representation of n and $m = 2$) or when $\mathbf{u}(n)$ is the residue of n modulo m. I'll also discuss in this context the Sarnak's conjecture which asserts than when the sequence $\mathbf{u}(\mathcal{A})$ is deterministic, i.e. when the number of blocks of length k is $\exp(o(k))$, the sequence $\mathbf{u}(\mathcal{A})$ is orthogonal to the Moebius function, this roughly meaning that $\sum_{n\le x} \mathbf{u}(a(n))\mu(n) = o(x)$ as x tends to infinity.

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